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VLF transmitter signals as a possible tool for detection of seismic effects on the ionosphere

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A R T I C L E   I N F O

A B S T R A C T

An overall consistent scheme is presented of using VLF transmitter signal spectral broadening observed on a satellite as a detection means of seismic activity. This includes the mechanisms for formation of small-scale plasma density irregularities, and generation of quasi-electrostatic lower hybrid resonance waves due to the scattering of transmitter signal from small-scale plasma irregularities. Both points are discussed in detail on quantitative level.

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1. Introduction

One of widely discussed seismic effects on the ionosphere is an excitation of small-scale plasma density inhomogeneities and correlated ELF emissions (see monographs edited by Hayakawa (1999), Hayakawa and Molchanov (2002), a review by Varotsos (2001) and references therein). First direct satellite observations of these phenomena were reported by Chmyrev et al. (1997), the generation mechanism in terms of dissipative instability of acoustic-gravity waves was proposed by Sorokin et al. (1998). Further development of the mechanism in the frame of electrodynamic model for the ionosphere response to seismic-related atmospheric disturbances is presented by Sorokin et al. (2001). Basing on this model, Sorokin et al. (2006) have suggested an idea to use the fixed frequency VLF transmitter signals as a tool for detection of small-scale plasma inhomogeneities presumably arising in the ionosphere before earthquakes.

The core of this idea is the well-known effect of spectral broadening of ground-based transmitter signals received on a satellite due to scattering from natural magnetic field-aligned plasma density irregularities and transformation of quasi-longitudinal whistler mode wave into quasi-electrostatic plasma wave (Bell et al., 1983; Titova et al., 1984; Tanaka et al., 1987; Bell and Ngo, 1988; Shklyar and Washimi, 1994). Owing to large wave-normal angles and short wavelengths, the phase velocities of these waves are low and comparable with the satellite velocity. Therefore, the Doppler shift becomes significant and the resultant signal received onboard a satellite displays a broadening of the spectrum. Similar effect caused by signal scattering on periodic plasma density inhomogeneities excited in the ionosphere by artificially injected modulated electron beam was reported by Oraevsky et al. (1994).

The objective of the present paper is the calculation of spectral broadening effect for various small-scale irregularities in seismically disturbed ionosphere, and evaluation of applicability of the above-mentioned idea to satellite monitoring of the ionospheric precursors to earthquakes.

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2. Generation mechanisms for seismic-related small-scale inhomogeneities in the ionosphere

One of possible mechanisms for modification of the ionosphere is connected with the formation in near-ground atmospheric layer of external electromotive force (EMF), which serves as a source of electric conductivity current in the global atmosphere–ionosphere circuit. This current flows vertically through the atmosphere, then in a conductive layer of the lower ionosphere and as a field-aligned current in the magnetosphere. Satellite observations of the electric field in magnetically conjugate zones of enhanced seismic and meteorological activity have been reported by Chmyrev et al. (1989), Isaev et al. (2002) and Sorokin et al. (2005a). Magnitudes of observed DC electric field disturbances were about 5–8 mV/m in the magnetic field tubes corresponding to earthquake area and 20–25 mV/m on the satellite passages over typhoon regions.

External electric current that causes the EMF is formed as a result of vertical turbulent transport of charged aerosols, their interaction with atmospheric ions at the injection of radioactive substances into the atmosphere and related modification of atmospheric conductivity (Sorokin et al., 2001, 2007). Some results of theoretical studies of external current effects on the ionosphere are given by Sorokin et al. (2005b) and Sorokin (2007). An important conclusion following from these papers is that the electric field amplification in the ionosphere and the formation of field-aligned electric current over the earthquake preparation zone lead to generation of plasma density inhomogeneities stretched along geomagnetic field lines in the upper ionosphere and the magnetosphere. Below, we consider some generation mechanisms for such inhomogeneities and estimate their characteristic spatial scales across the magnetic field.

2.1. Plasma redistribution caused by field-aligned current

One of well-known plasma stratification mechanisms is connected with generation of MHD waves in a zone of magnetic field-aligned currents in the ionosphere–magnetosphere system (Sato and Holzer, 1973). Propagation of ultra low-frequency waves in magnetized plasma is possible when transverse ($k_\perp$) and longitudinal ($k_\parallel = \omega/V_a$) components of the wave vector $\mathbf{k}$ satisfy the condition (Ginzburg, 1967)

$$k_\perp \ll k_\parallel \sqrt{\omega_e \omega_i / \omega \nu_e},$$

where ω_e, ω_i are the cyclotron frequencies of electrons and ions, ν_e is electron collision frequency and V_a is Alfven velocity. Assuming $V_a \approx 3 \times 10^5$ m/s; $\omega_e \omega_i \approx 3 \times 10^9$ s^{-2}; $\nu_e \sim 10^3$ s^{-1} in the upper ionosphere we find that $k_\parallel = 10^{-7}$ m^{-1} and $k_\perp \ll 10^{-3}$ m^{-1} for Alfven waves with frequency $\omega \leqslant 3 \times 10^{-2}$ s^{-1}. So the characteristic feature of field-aligned currents connected with MHD waves is their spatial modulation with characteristic scale $l_\perp = 2\pi/k_\perp \geqslant 6$ km across the magnetic field.

Other mechanism for plasma redistribution in F-region of the ionosphere is connected with stratification of field-aligned currents caused by fluctuations $\mathbf{E}_1$ of the electric field $\mathbf{E}_0$ (Kan and Okuda, 1983). Current density $\mathbf{j} = eN(\mathbf{v}_i - \mathbf{v}_e)$ is determined by the motion of ions and electrons in the ionosphere with velocities $\mathbf{v}_i$ and $\mathbf{v}_e$, respectively. Let us introduce the magnitudes of current $\langle \mathbf{j} \rangle$ and plasma density $\langle N \rangle$ in the ionosphere averaged over the spatial scale much larger than the characteristic scale of fluctuations. Current disturbances $\mathbf{j}^{(1)} = \mathbf{j} - \langle \mathbf{j} \rangle$ define plasma density variations $N_1 = N - \langle N \rangle$. In Cartesian coordinates (x,y,z) with z-axis directed upward along the magnetic field $\mathbf{B}$ the continuity equation for current disturbances $\nabla \cdot \mathbf{j}^{(1)} = 0$ can be written as

$$\frac{\partial j_\parallel^{(1)}}{\partial z} + \nabla_\perp \cdot j_\perp^{(1)} = 0.$$

Integration of this equation along z in the conductive layer of the lower ionosphere where transverse currents flow yields in linear approximation

$$j_\parallel^{(1)} = -\Sigma_P \nabla \cdot \mathbf{E}_1 - \int dz (\nabla_\perp \cdot N_1 \mathbf{u}_0),$$

where $\mathbf{u}_0 = \mathbf{v}_{i0} - \mathbf{v}_{e0}$ and Σ_P is the Pedersen integral conductivity. In a case of large-scale inhomogeneities $l_\perp \geqslant 10$ km this equation together with the continuity equation for ions gives

$$\beta N_1 - \frac{\partial}{\partial z}\left(D_0 \frac{\partial N_1}{\partial z} + \frac{g}{\nu_{in}}N_1\right) = -\frac{\sigma_P}{e\Sigma_P}j_\parallel^{(1)},$$

where β is a coefficient of recombination, D_0 is diffusion coefficient, g is gravitational acceleration, ν_{in} is ion-molecular collision frequency, and σ_P is Pedersen conductivity. This equation shows that layering of field-aligned currents leads to generation of plasma density inhomogeneities in F-region of the ionosphere. Negative fluctuations increase the density while the positive ones decrease it.

Transverse spatial size $l_\perp$ of small-scale plasma inhomogeneties connected with fluctuations of field-aligned currents can be found from the condition of equality of longitudinal and transverse resistances of the current circuit (Gel'berg, 1984). This circuit includes the field-aligned current of length L and the closing current in the lower ionosphere. Thus, we have

$$l_\perp = \sqrt{\frac{2\nu_{in}\nu_{ei}}{\omega_i \omega_e}HL},$$

where H is the characteristic altitude of inhomogeneous ionosphere. Assuming that $\nu_{ei} \sim 10^3$ s^{-1}; $\nu_{in} \sim 0.3$ s^{-1}; $H \approx 3 \times 10^4$ m; $L \approx 5 \times 10^6$ m at the altitude ~ 300 km, we obtain $l_\perp \sim 100$ m. Above maximum of F-layer where influence of recombination is small and the variations of density are determined by diffusion and the large-scale current, the relative amplitude of small-scale plasma density inhomogeneities can be estimated as

$$\frac{N_1}{N_0} \approx \frac{j_\parallel^{(1)}}{j_\parallel^{(0)}}\frac{H v_{e\parallel}^{(0)} \nu_{in}}{2c_s^2},$$

where $v_{e\parallel}^{(0)}$ is a thermal velocity of electrons carrying the field-aligned current, $c_s = \sqrt{(T_e + T_i)/M}$ is an ion sound velocity, T_e is electron temperature, T_i, M are temperature

and mass of ions. Assuming that $H \approx 3 \times 10^4$ m; $v_{e||}^{(0)} \approx 10^2$ m/s; $2c_s^2 \approx 10^6$ m^2 s^{-2}; $v_{in} \approx 0.3$ s^{-1}, we find:

$$N_1/N_0 \approx 0.9 j_{||}^{(1)}/j_{||}^{(0)}.$$

With increase of altitude the magnitude of plasma density disturbances decreases as $N_1 \propto N_0(z)v_{in}(z)$.

2.2. Generation of ionospheric plasma inhomogeneities caused by AGW instability in presence of external DC electric field

Sorokin et al. (1998) have shown that growth of DC electric field above some threshold level leads to instability of acoustic-gravity waves (AGW) in the ionosphere. This instability is connected with transformation of Joule heat of ionospheric currents into wave energy. Propagation of AGW in the ionosphere is accompanied by perturbation of the conductivity and, consequently, the currents. The source of energy for AGW instability is EMF of external electric field. In the case of horizontal propagation of a plane wave along the x-axis, the magnitudes of small perturbations of velocity $\mathbf{v}_1$, density ρ_1, pressure p_1, and temperature T_1 relative to their stationary values $\mathbf{v}_0, \rho_0, p_0, T_0$ in isothermal conductive and exponentially inhomogeneous ionosphere obey the equations

$$\rho_s(\partial/\partial t + \omega_m)U = -\partial P/\partial x;$$
$$\rho_s \partial W/\partial t = -\partial P/\partial z + P/2H - gR;$$
$$\partial R/\partial t + \rho_s(\partial U/\partial x + \partial W/\partial z - W/2H) = 0;$$
$$(\partial/\partial t + \omega_m)[(\partial/\partial t - \omega_2)P - a^2(\partial/\partial t + \omega_1)R + (\gamma - 1)g\rho_s W]$$
$$+ \omega_m \omega_2 P + a^2 \omega_m \omega_1 R = 0.$$

The equations above are written in variables defined as

$$U = (\rho_0/\rho_S)^{1/2}v_{1x}; \quad W = (\rho_0/\rho_S)^{1/2}v_{1z};$$
$$P = (\rho_0/\rho_S)^{-1/2}p_1;$$
$$R = (\rho_0/\rho_S)^{-1/2}\rho_1; \quad \rho_0 = \rho_s \exp(-z/H),$$

where ρ_s is the density of the atmosphere at some level corresponding to $z = 0$. In these equations, the following notation is used:

$$\omega_1 = (2\alpha + 1)(\gamma - 1)\sigma_{P0}E^2/2a^2\rho_0;$$
$$\omega_2 = \gamma(\gamma - 1)\sigma_{P0}E^2/2a^2\rho_0;$$
$$\omega_m = \sigma_{P0}B^2/c^2\rho_0; \quad a^2 = \gamma R T_0,$$

where γ is a specific heat ratio, R is the universal gas constant, and σ_{p0} is the Pedersen conductivity of undisturbed ionosphere. Coefficient α characterizes the relationship between relative variations of ion density and gas density in a wave. The quantity ω_1 has the sense of ratio of specific power of currents caused by the disturbances of ionospheric conductivity in the electric field and power density of acoustic wave. Its value defines the characteristic time during which the energy of external electric field transforms into kinetic energy of the wave. The value of ω_m characterizes the wave attenuation due to magnetic viscosity. Let the unknown variables be dependent on the coordinate and time as $\exp(-i\omega t + ikx)$. The set of equations above then yields the dispersion equation

$$\left(\frac{ak}{\omega}\right)^2$$
$$= \frac{(\omega + i\omega_m)\{\omega^2[\omega + i(\omega_m - \omega_2)] - (\omega + i\omega_m)\omega_a^2 + i\omega_2\omega_3^2\}}{\omega\{\omega^2[\omega + i(\omega_m + \omega_1)] - (\omega + i\omega_m)\omega_g^2\}},$$

where $\omega_a^2 = \gamma g/4H$ is the boundary acoustic frequency squared, $\omega_g^2 = (\gamma - 1)g/\gamma H$ is the Brunt–Vaisala frequency squared, $\omega_3^2 = \omega_a^2(2\alpha + 3)/\gamma$, $ka/\omega = (n + i\kappa)$; is the refraction index. Fig. 1 presents the results of calculations of the dependencies $n = n(\omega)$, $\kappa = \kappa(\omega)$. It is seen from the graphs that the absorption coefficient is negative and has the maximum absolute value at frequencies $\omega \sim \omega_g$. This means that the waves with frequency of the order of ω_g are exponentially growing and standing out the background in their amplitude forming a periodic structure. Along with oscillations of the density and pressure in the wave, the conductivity variations occur. Hence, the wave instability leads to formation of the horizontal periodic structure of ionospheric conductivity with the characteristic scale $l = \lambda/2$, where λ is the wavelength corresponding to $\omega \sim \omega_g$. At these frequencies the refractive index reaches its maximum value $n(\omega_g)$ corresponding to lowest phase velocity of the wave as compared with the sound velocity $v_g = a/n(\omega_g) < a$. The horizontal scale of the conductivity variation has the value

$$l = \lambda/2 = \pi v_g/\omega_g = \pi a/\omega_g n(\omega_g).$$
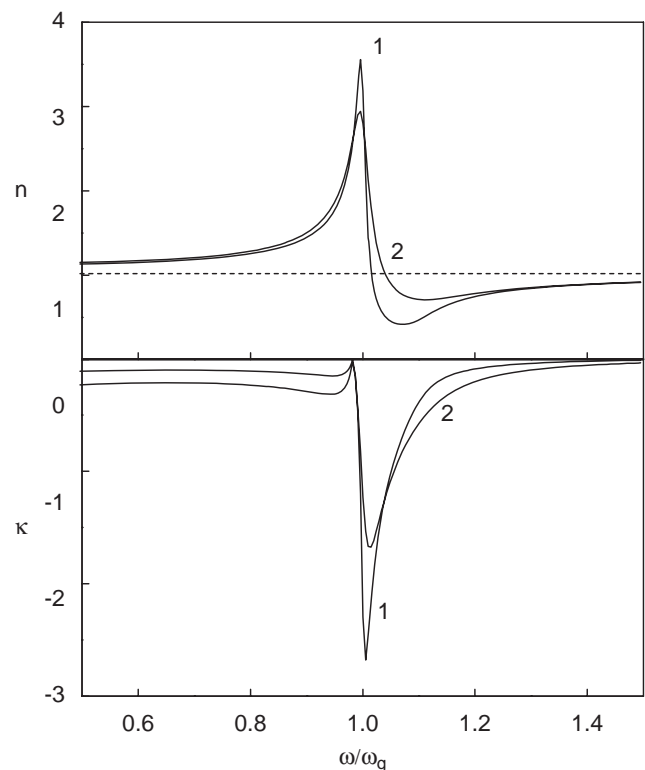
Fig. 1. Frequency dependence of the refractive index $n = n(\omega)$ (upper panel) and the absorption index $\kappa = \kappa(\omega)$ (lower panel) of AGW propagating horizontally in an electric field (Sorokin et al., 1998). The first curve corresponds to $\alpha = 0{,}2$ ($\omega_1/\omega_g = 1.4 \times 10^{-2}$; $\omega_3/\omega_g = 2.7$), and the second curve corresponds to $\alpha = 1$ ($\omega_1/\omega_g = 3 \times 10^{-2}$; $\omega_3/\omega_g = 3.9$).

Under the influence of external electric field, the polarization fields arise in horizontal irregularities of conductivity in the lower ionosphere. High conductivity along the magnetic field results in transferring the polarization field to the upper ionosphere and the magnetosphere. The electric circuit arising in this case includes the field-aligned currents and the closure currents due to the Pedersen conductivity. This process is accompanied by plasma density variations and the formation of plasma layers stretched along the magnetic field. If AGW propagates along the x-axis, the conductivity irregularities with spatial scale $l = \lambda/2$ are stretched along the y-axis. The electric field is directed along the x-axis. Horizontal velocity of the layers in x-direction coincides with the velocity of AGW and has the value $v_g = a/n(\omega_g, \omega_1)$. Integration along the x-axis of stationary continuity equation on the boundary of layer and determination of the electric field within the layer from the continuity condition of transverse ionospheric currents and field-aligned currents yield

$$\frac{N_1}{N_0} = \frac{v_i c E_{x0}}{2\omega_i v_g B} = \frac{v_i c n(\omega_g) E_{x0}}{2\omega_i a B}.$$

This formula enables one to evaluate the order of magnitude of plasma irregularities caused by the conductivity disturbances in E region of the ionosphere. This magnitude depends on altitude as $v_i = v_i(z)$. The transverse spatial scales of these irregularities coincide with the scales of the horizontal spatial structure of conductivity. Assuming that

$$v_i \approx 0.5\,\text{s}^{-1}; \quad \omega_i \approx 30\,\text{s}^{-1}; \quad a \approx 3 \times 10^2\,\text{m} \div \text{s};$$
$$E_0 \approx 9\,\text{mV/m} \approx 3 \times 10^{-7}\,cgse;$$
$$B \approx 0.3\,\text{G}; \quad \omega_g \approx 2 \times 10^{-2}\,\text{s}^{-1}; \quad n(\omega_g) \approx 1/10$$

we find $l_\perp \approx \pi a/\omega_g n(\omega_g) \approx (4 \div 40)\,\text{km}$; $N_1/N_0 \approx (1.6 \div 16)\%$. A satellite moving with the velocity $v_s \sim 10^4\,\text{m/s}$ through such plasma irregularities with the given above spatial scale will observe the plasma density fluctuations with a characteristic period $\tau \approx l_\perp/v_s \approx (0.4 \div 4)\text{s}$. As the irregularities are connected with caused field-aligned current, the satellite will also observe the ULF magnetic field oscillations with the same period. Their amplitude b is estimated as $b \approx (\pi/c)E\Sigma_P[\Delta\sigma_P/(2\sigma_{P0} + \Delta\sigma_P)] \approx 5nT$ that is in agreement with satellite data (Chmyrev et al., 1989).

Large transverse gradient of plasma density within the layers in presence of transverse electric field stimulates the development of instability and generation of small-scale irregularities. Gradient-drift instability together with drift-dissipative instability lead to the growth of waves propagating almost perpendicular to the geomagnetic field (Ried, 1968). Their combined growth rate at the anisotropy $k_\perp/k_\parallel \ll \sqrt{\omega_e\omega_i/v_e v_i} \sim 10^4$ has been found by Gel'berg (1981)

$$\gamma_G = \frac{k_\perp^2}{k_\parallel^2} \frac{v_e v_i}{\omega_e \omega_i}\left(\delta V_d + \frac{\delta^2 c_s^2}{v_i}\right),$$

where $\delta = N_0^{-1}\,dN_0/dx$ is determined by the plasma density gradient and $V_d \approx cE_0/B$ is the plasma drift velocity.

The inhomogeneity diffusion at such anisotropy is characterized by the growth rate $\gamma_D = D_0 k_\parallel^2$, where $D_0 = c_s^2/v_i$. The instability arises at the transverse scales defined by the inequality $\gamma_G \geqslant \gamma_D$, which yields

$$k_\perp^2 \geqslant k_\parallel^4 \frac{\omega_e \omega_i}{v_e v_i}\left/\left(\delta^2 + \frac{\delta v_i V_d}{c_s^2}\right)\right..$$

To estimate the transverse size of the resultant small-scale irregularities we assume $k_\parallel \approx 3 \times 10^{-5}\,\text{m}^{-1}$; $\omega_e\omega_i/v_e v_i \approx 10^8$; $\delta \approx 10^{-4}\,\text{m}^{-1}$; $V_d \approx 10^3\,\text{m/s}$; $c_s \approx 8 \times 10^2$ m/s; $v_i \approx 0.3\,\text{s}^{-1}$. It gives $k_\perp \geqslant 4 \times 10^{-2}\,\text{m}^{-1}$. Thus, the instabilities cause the layering of field-aligned plasma irregularities with initial transverse scale $\sim 10\,\text{km}$ into small-scale irregularities with $l_\perp \leqslant 2\pi/k_\perp \approx 150\,\text{m}$. Plasma density fluctuations reported by Chmyrev et al. (1997) from COSMOS-1809 satellite data had not included such small-scale irregularities due to limitations in data sampling rate on this satellite. So, the existence of plasma disturbances with characteristic transverse spatial scale of the order of or less than 100 m over the earthquake region, as predicted above, remains to be confirmed experimentally.

It is necessary to note that the development of AGW instability in the ionosphere leading to formation of significant spatial gradients of plasma density and allocated structure of field-aligned currents at certain gradient values causes the splitting of current layers and generation of filamentary structures with characteristic size across the magnetic field of the order of c/ω_{pe}. In the upper ionosphere this value is $\geqslant 100\,\text{m}$. The importance of these effects in applications to earthquake-related ionospheric disturbances was first noted by Chmyrev et al. (1999). In the high-latitude ionosphere, the effects of plasma and field-aligned current filamentation due to strong plasma density gradients was demonstrated by Chmyrev et al. (1988). Streltsov et al. (1990) have shown that the parallel current sheets or the solitary current disturbances in the auroral plasma are forming the vortex chains.

Thus, the calculations made in this section show that one can expect two characteristic spatial scales for seismic-related ionospheric disturbances: 4–40 km and $\sim 100\,\text{m}$ across the magnetic field. Direct measurements of such disturbances on the satellites and the related effects of VLF wave scattering on small-scale plasma density irregularities can be used for diagnostics of the ionosphere perturbations connected with earthquake preparation processes.

3. Effects of small-scale plasma inhomogeneities on the characteristics of VLF transmitter signals in the upper ionosphere over earthquake region

In this section, we describe the excitation of short wavelength, lower hybrid resonance (LHR) waves by VLF transmitter signal in presence of small-scale plasma density irregularities. As has been shown in Section 2, such irregularities may be formed in the near-Earth plasma in conjunction with earthquakes. As we will see below, the excited waves have the same frequency as the transmitter signal, but much larger transversal wave

number. When registered on a satellite, these waves may exhibit a significant bandwidth due to Doppler shift. Since the central frequency of the observed spectrum coincides with VLF transmitter frequency, this effect reveals as a spectral broadening of the transmitter signal.

As is well known, in the upper ionosphere, VLF transmitter signals propagate in whistler mode. For the sake of definiteness, in the following discussion we will limit ourselves to the case of cold dense plasma, i.e. we will assume the inequality $\omega_p^2 \gg \omega_c^2$ to be fulfilled, where ω_p and ω_c are electron plasma frequency and electron cyclotron frequency, respectively. In this case, whistler mode occupies the frequency band

$$\omega_{ci} \ll \omega \sim \, < \omega_c$$

and is described by the dispersion relation

$$\omega^2 = \omega_{LH}^2 \frac{k^2}{k^2 + q^2} + \omega_c^2 \frac{k_{\parallel}^2 k^2}{(k^2 + q^2)^2}$$
$$\equiv \frac{\omega_{LH}^2}{1 + q^2/k^2} + \frac{\omega_c^2 \cos^2 \theta}{(1 + q^2/k^2)^2}, \qquad (1)$$

where the lower hybrid resonance (LHR) frequency ω_{LH} is given by

$$\omega_{LH}^2 \simeq \frac{\omega_c^2}{M_{eff}}; \quad \frac{1}{M_{eff}} = \frac{m_e}{n_e} \sum_{ions} \frac{n_\alpha}{m_\alpha}, \qquad (2)$$

(n_e and m_e are the electron concentration and mass, respectively, while n_α and m_α are the same quantities for ions of the species α), $k^2 = k_{\parallel}^2 + k_{\perp}^2$, where $k_{\parallel}$ and $k_{\perp}$ are the components of the wave-normal vector parallel and perpendicular to the ambient magnetic field, respectively, $\theta = \cos^{-1}(k_{\parallel}/k)$, and

$$q^2 = \omega_p^2/c^2. \qquad (3)$$

From (1) one can see that the characteristic value of the wave number in the whistler frequency band is $q \equiv \omega_p/c$, and that for a given wave-normal angle θ, the dependence of the wave frequency on k involves only the ratio k^2/q^2.

In the frequency range indicated above, whistler waves represent the only plasma mode which may propagate in dense cold plasma, i.e., for given $k_{\parallel}$ and $k_{\perp}$ there is only one frequency corresponding to propagating wave. On the other hand, the same frequency may correspond to two distinctly different wave-normal vectors, or, more specifically, the same $k_{\parallel}$ and two different $k_{\perp}$ may be related to the same frequency ω. After all, this feature of whistler mode waves is behind the effects of LHR wave excitation by VLF transmitter signals in presence of small-scale plasma density irregularities considered below. This effects is similar to LHR wave excitation by whistlers. Both phenomena have been known for decades and discussed by several authors (Barrington et al., 1963; Brice and Smith, 1964; Jiricek and Triska, 1976; Titova et al., 1984; Bell and Ngo, 1988; Shklyar and Nagano, 1998). Different theoretical approaches to the problem under discussion were pointed out in the paper by Shklyar and Washimi (1994) devoted to the theory of LHR wave excitation by whistlers, and we adhere to this work in the following analysis.

Before going to quantitative estimations, we will describe the mechanism of LHR wave excitation qualitatively. A VLF transmitter signal propagating in the upper ionosphere as a whistler mode wave of frequency ω_{WH} and wave vector $\mathbf{k}_{WH}$ is associated with particle velocity oscillations of the corresponding frequency and wave vector. In a plasma with spatial density fluctuations these oscillations give rise to a plasma current with the wave vector

$$\mathbf{k} = \mathbf{k}_{WH} + \mathbf{k}_{INH}, \qquad (4)$$

where $\mathbf{k}_{INH}$ is the effective wave vector of plasma density fluctuations. Those harmonics of this current which are in resonance with plasma eigenmodes (in particular, LHR oscillations) excite waves which persist after the exciting wave escapes from the region where small-scale density irregularities are present.

Proceeding to quantitative description of the phenomena, we first notice that the dispersion relation (1) may return $\omega^2 \simeq \omega_{LH}^2$ in two domains of wave-normal vectors

$$k_{\parallel}^2 \leqslant k^2 \sim \frac{q^2}{\sqrt{M_{eff}}} \ll q^2,$$

which corresponds to dispersion relation of the form

$$\omega^2 = \omega_c^2 \frac{k_{\parallel}^2 k^2}{q^4} \qquad (5)$$

and

$$k^2 \gg q^2; \quad k_{\parallel}^2 \equiv k^2 \cos^2 \theta \leqslant \frac{k^2}{M_{eff}} \ll k^2,$$

where the dispersion relation takes the form

$$\omega^2 = \omega_{LH}^2 + \omega_c^2 \frac{k_{\parallel}^2}{k_{\perp}^2}. \qquad (6)$$

The first domain, and the corresponding dispersion relation (5) correspond to quasi-longitudinal electromagnetic waves. If describing the wave field with the help of quantities A_1, A_2, φ connected with the wave vector potential $A = (A_x, A_y, A_z)$ by the relations

$$A_x = A_1 - \frac{\partial \varphi}{\partial x}; \quad A_y = A_2; \quad A_z = -\frac{\partial \varphi}{\partial z}, \qquad (7)$$

then, in a homogeneous case and in the lowest approximation in parameter k^2/q^2, the quantity A_2 obeys differential equation (cf. the dispersion relation (5))

$$\frac{\partial^2 (\Delta A_2^{(0)})}{\partial z^2} + \frac{\overline{q^4}}{\omega_c^2} \frac{\partial^2 A_2^{(0)}}{\partial t^2} = 0. \qquad (8)$$

Here and further, we limit ourselves to consideration of 2D geometry, assuming the wave propagation in a meridian plane (x, z), with the ambient magnetic field directed along the z-axis. Thus, the Laplace operator which enters Eq. (8) has only two terms

$$\Delta = \frac{\partial}{\partial x^2} + \frac{\partial}{\partial z^2}.$$

We gave the quantity $A_2^{(0)}$ the superscript $^{(0)}$ to underline that it represents the zero-order unperturbed solution, and marked by bar the averaged constant value of the parameter q^4.

The second domain, and the related dispersion Eq. (6) corresponds to quasi-electrostatic waves, as it is usually the case for $k^2 \to \infty$. These waves are described by the above-introduced quantity φ which is connected with the wave scalar potential Φ by the relation

$$\Phi = -c^{-1}\partial\varphi/\partial t.$$

Because of this simple relation between Φ and φ, the latter will also be referred to by potential for shortness. As has been shown by Shklyar and Washimi (1994), in variables φ and $A_2^{(0)}$, and in the first non-vanishing approximation, the process of LHR wave excitation by whistlers is described by

$$\frac{\partial}{\partial x}\left[\left(\bar{\alpha}\frac{\partial^2}{\partial t^2} + \bar{\beta}\right)\frac{\partial\varphi}{\partial x}\right] + \frac{\partial}{\partial z}\left(\bar{\eta}\frac{\partial\varphi}{\partial z}\right) = -\frac{\partial}{\partial x}\left(\gamma\frac{\partial A_2^{(0)}}{\partial t}\right). \qquad (9)$$

In writing Eq. (9), the following notation has been used:

$$\alpha = \frac{\omega_p^2}{c^2\omega_c^2} \equiv \frac{q^2}{\omega_c^2}; \quad \beta = \omega_{LH}^2\frac{\omega_p^2}{c^2\omega_c^2} \equiv \frac{q^2}{M_{eff}};$$

$$\gamma = \frac{\omega_p^2}{c^2\omega_c} \equiv \frac{q^2}{\omega_c}; \quad \eta = \frac{\omega_p^2}{c^2} \equiv q^2, \qquad (10)$$

where the quantity q^2 is defined in (3), and, as above, the bar marks constant averaged value of the corresponding parameter. We see that the excitation of quasi-electrostatic LHR waves by quasi-longitudinal whistlers, in particular, by VLF transmitter signals, is caused by the fluctuation of the quantity $\gamma \propto \omega_p^2/\omega_c$.

Eq. (9) has been studied in detail by Shklyar and Washimi (1994). Using the method of Green function, they have investigated the case of two-dimensional inhomogeneity under assumption that the exciting field represents a wave packet of general form. Here, we will analyze Eq. (9) for particular case when the exciting field is that of monochromatic VLF transmitter signal. A simplification that we will adopt consists in the assumption of one-dimensional inhomogeneity; more specifically, we will assume that the quantity γ, which provides the coupling between two branches (see (9)) does not depend on z, i.e. that plasma density irregularities are predominantly elongated along the ambient magnetic field. This natural simplification does not invalidate the consideration, but makes it much more obvious. Introducing dimensionless variables

$$x' = (\bar{\eta})^{1/2}x; \quad z' = (\bar{\beta})^{1/2}z; \quad t' = \bar{\omega}_{LH}t \qquad (11)$$

we get from (9) (omitting primes)

$$\frac{\partial^4\varphi}{\partial x^2\partial t^2} + \frac{\partial^2\varphi}{\partial x^2} + \frac{\partial^2\varphi}{\partial z^2} = -\sigma\frac{\partial}{\partial x}\left(\gamma\frac{\partial A_2^{(0)}}{\partial t}\right); \quad \sigma \equiv (\alpha\beta\eta)^{-1/2}. \qquad (12)$$

Along with dimensionless time and coordinates, in which Eq. (12) is written, we will further use dimensionless frequency and wave-normal vector components

$$\omega' = \omega/\bar{\omega}_{LH}; \quad k'_x = k_x/\sqrt{\bar{\eta}}; \quad k'_z = k_z/\sqrt{\bar{\beta}} \qquad (13)$$

also omitting primes in analytical calculations.

We will write the exciting field $A_2^{(0)}$ in the form

$$A_2^{(0)} = \text{Re}[A_0\exp(i\mathbf{k}_0\mathbf{r} - i\omega_0 t)], \qquad (14)$$

where ω_0 and $\mathbf{k}_0$ are transmitter signal frequency and wave-normal vector, respectively, which are related by the dispersion Eq. (5). Similar form will be used for the quantity φ

$$\varphi = \text{Re}\{\tilde{\phi}\}. \qquad (15)$$

Obviously, if the quantity $\tilde{\phi}$ satisfies Eq. (12) with $A_2^{(0)}$ substituted by $A_0\exp(i\mathbf{k}_0\mathbf{r} - i\omega_0 t)$, i.e.

$$\frac{\partial^4\tilde{\phi}}{\partial x^2\partial t^2} + \frac{\partial^2\tilde{\phi}}{\partial x^2} + \frac{\partial^2\tilde{\phi}}{\partial z^2} = -\sigma\frac{\partial^2}{\partial x\partial t}[\gamma A_0\exp(i\mathbf{k}_0\mathbf{r} - i\omega_0 t)], \qquad (16)$$

then φ will satisfy Eq. (12), because the coefficients σ and γ are real.

Since the right-hand side of the equation for $\tilde{\phi}$ is proportional to $\exp(ik_{0z}z - i\omega_0 t)$, the same dependence on time and coordinate z may be used for $\tilde{\phi}$, i.e., we may look for a solution in the form

$$\tilde{\phi} = \phi(x)\exp(ik_{0z}z - i\omega_0 t). \qquad (17)$$

Substituting (17) into (16) we come to the ordinary second-order differential equation for the quantity ϕ

$$\frac{\partial^2\phi}{\partial x^2} + \kappa^2\phi = -\frac{i\omega_0\sigma A_0}{(\omega_0^2 - 1)}\frac{\partial}{\partial x}[\gamma\exp(ik_{0x}x)]; \quad \kappa^2 \equiv \frac{k_{0z}^2}{\omega_0^2 - 1}. \qquad (18)$$

The quantity κ introduced above should not be mixed with imaginary part of the refraction index $\kappa(\omega)$ which appeared in the previous section.

We see that Eq. (18) has solutions of the wave type ($\kappa^2 > 0$) only for $\omega_0 > 1$, i.e. when the transmitter frequency is above the LHR frequency. The quantity κ has a simple physical meaning: it is the solution (with respect to k_x) of the dispersion relation (6) for given frequency ω_0 and longitudinal wave number k_{0z}. Obviously, for given ω_0 and k_{0z}, there are two possible transversal wave numbers of different signs. For the sake of definiteness, we will assume that $\kappa > 0$, i.e.

$$\kappa = \sqrt{\frac{k_{0z}^2}{\omega_0^2 - 1}}. \qquad (19)$$

Eq. (18) represents the linear pendulum equation with a driving force, and its general solution is well known. The particular one, which describes the excitation of LHR waves by VLF transmitter signal and satisfies appropriate boundary conditions (see below) has the form

$$\phi = -\frac{i\omega_0\sigma A_0\kappa^2}{2k_{0z}^2}\left\{e^{-i\kappa x}\int_{-\infty}^{x}\gamma(x')e^{i(k_{0x}+\kappa)x'}\,dx'\right.$$
$$\left. - e^{i\kappa x}\int_{x}^{\infty}\gamma(x')e^{i(k_{0x}-\kappa)x'}\,dx'\right\}. \qquad (20)$$

Outside the excitation region, the first term in braces in (20) describes the wave with a negative transversal wave number. Since this wave has a positive transversal group velocity (see (6)), the corresponding term should vanish at $x \to -\infty$. For the second term, the situation is reverse. This

determines the limits of integration in the corresponding integrals. Using relations (15), (17)), and the explicit form (19) for κ, we obtain from (20) the expression for potential φ

$$\varphi = \frac{\omega_0 \sigma A_0}{2(\omega_0^2 - 1)} \mathrm{Im} \left\{ \exp\left(-i\omega_0 t + ik_{0z}z - \frac{ik_{0z}x}{\sqrt{\omega_0^2 - 1}} \right) \right.$$

$$\int_{-\infty}^{x} \gamma(x') \exp\left[i\left(k_{0x} + \frac{k_{0z}}{\sqrt{\omega_0^2 - 1}} \right) x' \right] dx'$$

$$- \exp\left(-i\omega_0 t + ik_{0z}z + \frac{ik_{0z}x}{\sqrt{\omega_0^2 - 1}} \right)$$

$$\left. \int_{x}^{\infty} \gamma(x') \exp\left[i\left(k_{0x} - \frac{k_{0z}}{\sqrt{\omega_0^2 - 1}} \right) x' \right] dx' \right\}. \quad (21)$$

Far outside from the excitation region, i.e. at $x \to \pm \infty$, expression (21) is reduced to

$$\varphi_\pm = \pm \frac{\omega_0 \sigma A_0}{2(\omega_0^2 - 1)} \mathrm{Im} \left\{ \exp\left(-i\omega_0 t + ik_{0z}z \mp \frac{ik_{0z}x}{\sqrt{\omega_0^2 - 1}} \right) \right.$$

$$\left. \times \tilde{\gamma}\left(-k_{0x} \mp \frac{k_{0z}}{\sqrt{\omega_0^2 - 1}} \right) \right\}, \quad (22)$$

where $\tilde{\gamma}(k)$ is the Fourier transform of the quantity $\gamma(x)$ corresponding to the wave number k

$$\tilde{\gamma}(k) = \int_{-\infty}^{\infty} \gamma(x') e^{-ikx'} dx'. \quad (23)$$

Thus, the efficiency of the LHR wave excitation is determined by the Fourier component of the quantity $\gamma(x)$ at at $k = k_{INH}$, where

$$k_{INH} = -k_{0x} \mp \frac{k_{0z}}{\sqrt{\omega_0^2 - 1}}. \quad (24)$$

Expression (22) clearly shows that the frequency and the wave-normal vector of the excited wave satisfy the dispersion relation (6) for quasi-electrostatic LHR waves, and that the following equalities hold:

$$k_{zLH} = k_{0z}; \quad k_{xLH} = k_{0x} + k_{INH}; \quad \omega_{LH} = \omega_0, \quad (25)$$

where the subscript "LH" marks the quantities related to the excited LHR wave, and subscript "0", as before, is related to transmitter signal. Since under accepted simplification $k_{zINH} = 0$, the first two relations in (25) may be written as $\mathbf{k}_{LH} = \mathbf{k}_0 + \mathbf{k}_{INH}$. We should mention that expression (22) for φ does not tend to infinity when $\omega_0 \to 1$, i.e. when transmitter frequency approaches the LHR frequency, since, together with the denominator, the quantity $\tilde{\gamma}(k_{INH})$ also tends to zero as $k_{INH} \to \infty$.

As for the characteristics of the excited spectrum, we may say the following. Both the spectral maximum and the spectral width are determined by the ratio $\tilde{\gamma}(k_{INH})/(\omega_0^2 - 1)$. If the quantity $\tilde{\gamma}(k_{INH})$ does not have a sharp maximum and smoothly goes to zero at large

k_{INH}, then the spectrum of excited waves will have a pronounced maximum near the LHR frequency and drop as $\omega_{LH}/(\omega_0 - \omega_{LH})$ with increasing frequency ω_0.

Thus, in the upper ionosphere over the earthquake region, where small-scale plasma density irregularities are present, we should expect excitation of quasi-electrostatic LHR waves by transmitter signals, the excitation being most pronounced in the region where the transmitter frequency is above, but close to the local LHR frequency. These waves, being eigen modes of the ionospheric plasma, will propagate outside the excitation region and may be observed by satellites. Since the excited LHR waves have large values of wave-normal vector, when observed on satellites they should exhibit a noticeable Doppler shift of the order $\mathbf{k}_{LH} \cdot \mathbf{v}_{sat} \simeq \mathbf{k}_{INH} \cdot \mathbf{v}_{sat}$, where $\mathbf{v}_{sat}$ is a typical satellite velocity. Assuming that the small-scale inhomogeneities have typical dimension across the magnetic field of the order of a few tens of meters, and that $\mathbf{k}_{INH}$ is of the order of inverse inhomogeneity scale, we get the spectral broadening $\sim 100\,\mathrm{Hz}$.

4. Conclusions

In this paper, we have analyzed the mechanisms for generation of small-scale plasma density irregularities in the ionosphere over the seismic zone, and the effects of these irregularities upon characteristics of VLF transmitter signals propagated through these disturbances and then registered onboard a satellite. The main effect consists in observable spectral broadening of signals. The calculations have given two characteristic spatial scales of plasma density irregularities across the magnetic field. The first is 4–40 km, which has been confirmed by satellite observations, and the second is of the order or less than 100 m. These smaller size irregularities produce noticeable effect in VLF signal spectral broadening, which is most pronounced in the case when the transmitter frequency is above, but close to the local LHR frequency in the region where the small-scale irregularities are present, which sets the requirement that the VLF transmitter frequency be in the range from 6 to 12 kHz. This corresponds to operational band of most VLF transmitters. For the 100 m irregularities we get the spectral broadening $\sim 100\,\mathrm{Hz}$ that can easily be registered by simple VLF receiver onboard a satellite, provided that VLF transmitter power is high enough. (Radiated power of existing VLF transmitters is from tens to hundreds kW.) This effect together with the direct satellite measurements of plasma density variations can be used as an effective tool for diagnostics of seismic-related ionospheric disturbances and therefore considered as a possible ionospheric precursor to earthquake.

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