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On the Stress-Strain State of Film Structures in Space at Electrodynamic Deployment

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Abstract—We discuss a possibility of using the Ampere force for deployment and stabilization of film structures in space. It is supposed that a conductor connected with a power supply is deposited on the surface of a thin film, or solar batteries are used as conductors with current. The stress and strain of the thin film, caused by the electric current flow over a circular conductor confining this film, are found. The method of calculating the radial component of the stress tensor in a film, connected with concentric circular conductors with current, is obtained. The distribution of stresses for a film with three conductors is calculated. A possibility of minimizing the magnetic moment of a system of currents is discussed.

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1. INTRODUCTION

The increase of efficiency of solar batteries used on satellites is one of topical problems. In order to solve this problem, it is required to increase the specific power of the solar battery, to improve its structure, and to reduce its weight [4]. Such requirements to solar batteries are claimed at developing of both small satellites and large space vehicles. In the future, highly efficient solar batteries will be required for space power plants. Currently, in manufacturing solar batteries the crystal silicone with a specific power of 40 W/kg and gallium arsenide with 80 W/kg are used. The specific power should increase by more than a factor of ten, when the thin film with the amorphous silicone, deposited on it, is used as a solar battery. This technology was suggested recently [5]. An obstacle for manufacturing solar batteries based on thin films was the absence of sufficiently efficient and technologically effective structures for their deployment and turning during the flight. The way out of this situation is the use of flexible space structures, whose development has been considerably advanced in Russia in recent years [6]. There are two promising methods of flexible structure deployment in space, which are being actively developed now. The first of them is based on using the centrifugal forces, and the second one is based on using the Ampere force for deployment and stabilization of film structures in space. The present paper is devoted to the analysis of the second method.

2. RADIAL STRESS IN A ROUND PLATE CONFINED BY THE CONDUCTOR WITH CURRENT

We consider a thin round plate of radius R and thickness h. The conductor having circular cross section of radius a is located over the circle at the edge of a nonconducting plate. The direct current I flows over the conductor and generates in an ambient space the magnetic field with induction $\mathbf{B} = \mu_0\mathbf{H}$. According to the Ampere law, this magnetic field results in appearance of the force acting on the conductor with current that generates this field. The component of force F_q, acting in the direction of coordinate axis q, is determined by the free energy of the magnetic field $\Psi(q, I)$ by the formula:

$$F_q = \left(\frac{\partial\Psi(q, I)}{\partial q}\right)_I. \qquad (1)$$

The free energy is determined by the magnetic field of current: $\Psi = \frac{1}{2}\int \mathbf{H}\cdot\mathbf{B}(dV)$, where integration is performed over the volume occupied by the field. Making use of the Maxwell equation $\nabla\times\mathbf{H} = \mathbf{j}$ and introducing the vector potential of the field according to the formula $\mathbf{B} = \nabla\times\mathbf{A}$, we obtain: $\Psi = \frac{1}{2}\int \mathbf{j}\cdot\mathbf{A}dV$. The vector potential is determined by the density of electric current $\mathbf{j}$ generating the magnetic field:

$$\mathbf{A} = \frac{\mu_0}{4\pi}\int\frac{\mathbf{j}}{R_j}dV,$$

where R_j is the distance between the current's element and an observation point. Making use of this expression for the vector potential, we obtain the formula for free energy of the magnetic field, generated by a thin closed conductor, in the form:

$$\Psi = L\frac{I^2}{2}; \quad L = \frac{\mu_0}{4\pi}\oint\oint\frac{d\mathbf{I}d\mathbf{I}'}{R_j}, \qquad (2)$$

where L is the coefficient of self-induction of a closed conductor. In deriving formula (2) the electric current density was replaced by the full current flowing in the conductor: $\mathbf{j}dV = Id\mathbf{I}$. In this formula $d\mathbf{I}$ and $d\mathbf{I}'$ are arbitrary elements of the conductor, the distance between which equals R_j. Substituting (2) into (1), we obtain the component of the force acting on a closed conductor with current:

$$F_q = \frac{I^2}{2}\frac{\partial L}{\partial q}. \qquad (3)$$

The self-induction coefficient (see Appendix, formula (A4)), has the form:

$$L = \mu_0 R\left(\ln\frac{8R}{a} - \frac{7}{4}\right). \qquad (4)$$

Substituting (4) into (3) and letting the generalized coordinate to be $q = 2\pi R$, we obtain the component of force $F_{\parallel}$ acting along the conductor's axis:

$$F_{\parallel} = \frac{\mu_0}{4\pi}I^2\left(\ln\frac{8R}{a} - \frac{3}{4}\right).$$

Consider now the element of a circular conductor leaning on the angle φ, whose length equals l. As is shown in Fig. 1, the force $F_{\perp}$, directed normally to the conductor and located in its plane, is the resultant of forces $F_{\parallel}$ applied to the edges of a selected element. Therefore, $F_{\perp} = 2F_{\parallel}\sin(\varphi/2)$. The force, normalized with respect to the unit of conductor's length $f_A = F_{\perp}/l = F_{\perp}/2R\sin(\varphi/2) = F_{\parallel}/R$, is determined by the formula:

$$f_A = \frac{\mu_0 I^2}{4\pi R}\left(\ln\frac{8R}{a} - \frac{3}{4}\right). \qquad (5)$$

Now we find the radial component of the stress tensor σ_{rr} in a plate. Stresses arise as a result of applying the Ampere force f_A to the plate's edges. We introduce the Cartesian coordinate system with axis z, directed normally to the plane of a plate located in the x, y plane. Under an effect of applied force, the strain $\mathbf{u}$ arises in the plate, which satisfies the equation [3]:

$$\frac{1-\sigma}{2}\Delta\mathbf{u} + \frac{1+\sigma}{2}\nabla(\nabla\mathbf{u}) = -\mathbf{P}\frac{1-\sigma^2}{Eh}, \qquad (6)$$

where σ is the Poisson coefficient, E is the Young's modulus, $\mathbf{P}$ is the density of volume forces normalized with respect to the unit of plate's area. In a resting plate, in the absence of gravity force, $\mathbf{P} = 0$. In the cylindrical coordinate system $r = \sqrt{x^2 + y^2}$; $\varphi = \arctan(y/x)$, equation (6) is as follows:

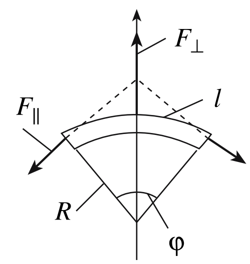

Fig. 1. A scheme illustrating the balance of forces applied to a separate element of a conductor.

$$\frac{d}{dr}\left[\frac{1}{r}\frac{d}{dr}(ru_r)\right] = 0.$$

This equation, under the condition $u_r(r = 0) = 0$, has the solution: $u_r = c_1 r$. The radial component of the stress tensor σ_{rr} is determined by the equality [3]:

$$\sigma_{rr} = \frac{E}{1-\sigma^2}\left(\frac{du_r}{dr} + \sigma\frac{u_r}{r}\right). \qquad (7)$$

Substituting $u_r = c_1 r$ into (7), we get: $\sigma_{rr} = c_1\frac{E}{1-\sigma}$.

Since the radial stress does not depend on radius, its value is determined by the radial component of the Ampere force f_A, applied to the edge of a plate, normalized with respect to the unit of conductor's length $\sigma_{rr} = f_A/h$. Therefore, $c_1 = f_A(1-\sigma)/Eh$. This value of the constant allows one to obtain the formulas for stress and strain of a thin plate confined by a circular conductor with current:

$$\sigma_{rr} = \frac{\mu_0 I^2}{4\pi Rh}\left(\ln\frac{8R}{a} - \frac{3}{4}\right);$$
$$u_r = \frac{\mu_0 r(1-\sigma)I^2}{4\pi REh}\left(\ln\frac{8R}{a} - \frac{3}{4}\right). \qquad (8)$$

Let us estimate the value of the radial component of the stress tensor. Letting in formula (8): $R = 1$ m, $h = 10^{-6}$ m, $a = 10^{-4}$ m, $I = 3A$, we obtain: $\sigma_{rr} = 12$ N/m^2. The value of the radial component of the force applied to the plate's edge, normalized with respect to the unit of the circle's length, is equal to $f = 1.2 \cdot 10^{-5}$ N/m. The presented estimates indicate that the ring electric current generates in a plate the stress whose tensor radial component does not depend on r. An increase of the value of the component of the stress tensor by two–three orders of magnitude is possible in the case, when the concentric circular currents are arranged on a plate in place of a single current loop considered above.

3. FORCES OF INTERACTION OF CIRCULAR CONCENTRIC CONDUCTORS WITH CURRENT ON A THIN PLATE

Now let us find the forces acting on a plate, on which thin conductors in the form of N concentric circles with radii r_i are arranged, as shown in Fig. 2. The numbers of circles assume the values $i = 1, 2, \ldots, N$. The greatest radius of the ring current coincides with the plate's radius $r_N = R$. Direct electric currents I_i flow over conductors and generate in the surrounding space the magnetic field with induction $\mathbf{B}$. The force $\Delta\mathbf{F}_i$ acting from the magnetic field side on current's element of the ith conductor $I\Delta d\mathbf{I}_i$ is determined by the expression: $\Delta\mathbf{F}_i = I\Delta\mathbf{I}_i \times \mathbf{B}$. The magnetic field, acting upon this conductor, is formed by currents in conductors $j \neq i$. In the (x, y) plane the field component $B_z = B$ is nonzero. The radial component of the Ampere force, acting upon the element of the ith conductor Δl_i, is determined by the formula:

$$\Delta F_i = I\Delta l_i B(r = r_i, z = 0).$$

The magnetic field $B(r = r_i, z = 0)$ is equal to the sum of fields, generated by all circular conductors except for the intrinsic field of the ith conductor. The magnetic field B_{ij} of the ring current I_j of radius r_j at the point with coordinates $z = 0$, $r = r_i$ has only the z-component, which is determined by the φ-component of the vector potential: $B_{ij} = \frac{1}{r}\frac{\partial}{\partial r}(rA_\varphi)_{r = r_i}$. The component of the vector potential A_φ) of the magnetic field of current I_j, flowing in a closed circular conductor of radius r_j, is determined by the formula:

$$A_\varphi = \frac{\mu_0 I_j}{4\pi}\oint\frac{\cos\varphi dl}{R_j}$$

$$= \frac{\mu_0 I_j}{2\pi}\int_0^\pi \frac{R\cos\varphi dl}{\sqrt{r_j^2 + r^2 + z^2 - 2r_j r\cos\varphi}},$$

where the angle φ is measured from the plane passing through the axis z and the field observation point. Introducing the new variable of integration $\theta = (\varphi - \pi)/2$, we obtain the z-component of the magnetic field in a conductor of radius r_i, located on the plane $z = 0$, in the form:

$$B_{ij} = \frac{\mu_0 I_j}{2\pi}\left[\frac{1}{r_j + r_i}K(\lambda_{ij}) + \frac{1}{r_j - r_i}E(\lambda_{ij})\right];$$

$$\lambda_{ij} = \frac{4r_i r_j}{(r_i + r_j)},$$

where $K(x)$ and $E(x)$ are the Legendre's complete elliptic integrals determined by the equalities [1]:

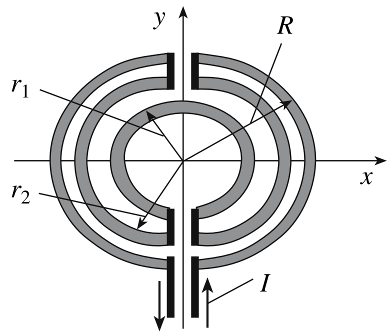

Fig. 2. One possible version of connection of circular concentric conductors on a thin plate.

$$K(\lambda_{ij}) = \int_0^{\pi/2}\frac{d\theta}{\sqrt{1 - \lambda_{ij}^2\sin^2\theta}};$$

$$E(\lambda_{ij}) = \int_0^{\pi/2}\sqrt{1 - \lambda_{ij}^2\sin^2\theta}d\theta.$$

The total magnetic field of all currents flowing in the conductors $j \neq i$, acting on the ith conductor, is equal to:

$$B(r = r_i, z = 0) = \sum_{j \neq i} B_{ij}.$$

Therefore, the radial component of the force f_i, acting upon a unit of length of the ith conductor located on a thin plate, is determined by the expression:

$$f_i = \frac{\Delta F_i}{\Delta l} = \frac{\mu_0 I_i}{2\pi}\sum_{j \neq i} I_j\left[\frac{1}{r_j + r_i}K(\lambda_{ij}) + \frac{1}{r_j - r_i}E(\lambda_{ij})\right]. \quad (9)$$

The surface density of forces, acting on a plate from the side of a set of thin concentric circular conductors with current, is equal to the sum:

$$P = \sum_i f_i\delta(r - r_i), \quad (10)$$

where $\delta(x)$ is the Dirac delta-function.

4. STRESSES IN A PLATE AT INTERACTION OF CIRCULAR CONCENTRIC CONDUCTORS WITH CURRENT

Let us find the stress tensor in a plate, on which thin conductors with electric current, forming the surface density of forces (10), are located. According to the Ampere law, the magnetic field results in the appearance of forces of interaction between conductors. Since the conductors are rigidly connected with the plate, these forces are transferred to the plate causing stresses in it. We should make calculation of strain and stress fields in a plate, taking into account only the forces of interaction of various conductors and neglecting the effect of intrinsic field on a conductor. Under the action of an applied force, the strain **u** arises in the plate, which satisfies equation (6). When axial symmetry takes place, this equation is as follows:

$$\frac{d}{dr}\left[\frac{1}{r}\frac{d}{dr}(ru_r)\right] = -P\frac{1-\sigma^2}{Eh}. \tag{11}$$

The component σ_{rr} of the stress tensor is determined by equality (7). Beyond the field of action of volume forces $r \neq r_i$ and $P = 0$, the radial component of strain $u_r = u(r)$ satisfies the homogeneous equation:

$$\frac{d}{dr}\left[\frac{1}{r}\frac{d}{dr}(ru_r)\right] = 0, \tag{12}$$

whose general solution is:

$$u = c_1 r + \frac{c_2}{r}, \tag{13}$$

where $c_{1,2}$ are integration constants. Their values are determined by the boundary conditions imposed on function $u(r)$ and its derivative at points $r = r_i$ of application of the Ampere forces. At these points the strain $u(r)$ is continuous, while du/dr bears discontinuity, whose value can be found by integration of equation (11) over the r vicinity of point $r = r_i$. As a result, we obtain a set of boundary conditions for the radial components of a strain vector:

$$u(r = r_i + 0) - u(r = r_i - 0) = 0;$$
$$\frac{\partial u(r = r_i + 0)}{\partial r} - \frac{\partial u(r = r_i - 0)}{\partial r} = -\frac{1-\sigma^2}{Eh} f_i = -g_i; \tag{14}$$
$$i = 1, 2, \ldots, N-1.$$

At the coordinate origin $r = 0$ the strain $u(r)$ should be bounded, and at the plate's edge $r = R$ the Ampere force, acting on the Nth conductor, should be compensated by the radial stress of the membrane: $\sigma_{rr}(r = R) = f_N/h$. Making use of equation (7), we obtain the conditions, to which the strain at membrane's center and boundary satisfies:

$$u(r = 0) < \infty; \quad \left(\frac{du(r = R)}{dr} + \sigma\frac{u(r = R)}{r}\right) = g_N. \tag{15}$$

Solution (13) of equations (11) with right-hand side (10) can be written as:

$$u(r) = k_i r + \frac{p_i}{r}; \quad r_i < r < r_{i+1}, \quad i = 0, \ldots, N-1. \tag{16}$$

Constants k_i, p_i in equality (16) are determined from boundary conditions (14) and (15). Having substituted (16) into (14), we obtain the recurrence relations for determination of the constants:

$$k_i - k_{i-1} = -\frac{1}{2}g_i; \quad p_i - p_{i-1} = \frac{1}{2}g_i r_i^2.$$

It follows from the boundedness of function $u(r)$ at $r = 0$, that $p_0 = 0$; therefore,

$$k_n = k_0 - \frac{1}{2}\sum_{i=1}^{n} g_i; \quad p_n = \frac{1}{2}\sum_{i=1}^{n} g_i r_i^2. \tag{17}$$

For determining k_0 one should use boundary condition (15), which gives the relation:

$$k_{N-1} - \frac{p_{N-1}}{R^2} + \sigma\left(k_{N-1} + \frac{p_{N-1}}{R^2}\right) = g_N.$$

Making use of equalities (17), we get:

$$k_n = \frac{g_N}{1+\sigma} + \frac{1-\sigma}{2(1+\sigma)R^2}\sum_{i=1}^{N-1} g_i r_i^2 + \frac{1}{2}\sum_{i=n+1}^{N-1} g_i;$$
$$p_0 = 0; \quad p_n = \frac{1}{2}\sum_{i=1}^{n} g_i r_i^2; \quad n = 0, \ldots, N-1. \tag{18}$$

Thus, the radial component of a strain vector is determined by the formula:

$$u(r) = k_i r + \frac{p_i}{r};$$
$$r_i < r < r_{i+1}, \quad 0 = r_0 < r_1 < \ldots < r_{N-1} < r_N = R,$$

where the constants k_i and p_i are determined by expressions (18), and the quantities g_i and λ_{ij} are found from the equalities:

$$g_i = \frac{1-\sigma^2}{Eh}\frac{\mu_0 I_i}{2\pi}\sum_{j \neq i} I_j\left[\frac{1}{r_j + r_i}K(\lambda_{ij}) + \frac{1}{r_j - r_i}E(\lambda_{ij})\right],$$
$$\lambda_{ij} = \frac{4 r_i r_j}{(r_i + r_j)^2}. \tag{19}$$

The radial component of a stress tensor, according to formula (7), is determined by the expression:

$$\sigma_{rr} = \frac{E}{1-\sigma^2}\left[(1+\sigma)k_i - (1-\sigma)\frac{p_i}{r^2}\right],$$
$$r_i < r < r_{i+1}. \tag{20}$$

It should be noted that the radial component of the stress tensor σ_{rr} bears discontinuities at the point $r = r_i$.

5. DISTRIBUTION OF STRESSES IN A PLATE WITH CIRCULAR CONCENTRIC CURRENTS

Except for the neighboring conductors, each circular current is affected by the Ampere force caused by the magnetic field generated by the same current. In this

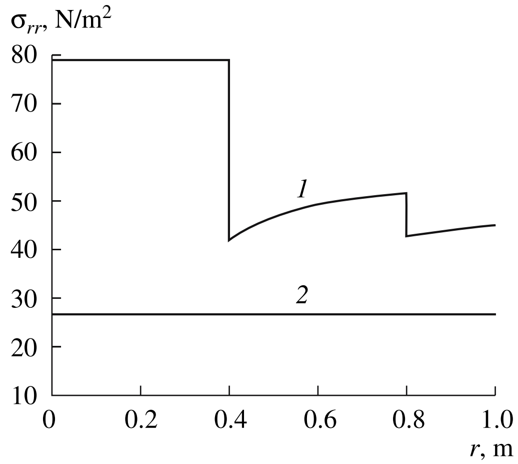

Fig. 3. The plot of a radial component of the stress tensor σ_{rr} in a thin plate with three concentric circular conductors (curve *1*) as a function of radius. The value of the radial component of the stress tensor in a thin plate with circular current (straight line *2*).

case, the radial component of the Ampere force ΔF_i, acting upon the element of the ith conductor, is determined by the formula:

$$\Delta F_i = \Delta F_{i0} + I_i \Delta l B(r = r_i, z = 0),$$

where ΔF_{i0} is the force, acting upon a conductor, caused by its intrinsic magnetic field, while the magnetic field $B(r = r_i, z = 0)$ is equal to the sum of fields generated by all remaining circular conductors. The force ΔF_{i0} is found from formula (5):

$$\Delta F_{i0} = \frac{\mu_0 I_i^2}{4\pi r_i}\left(\ln\frac{8r_i}{a} - \frac{3}{4}\right)\Delta l_i.$$

Adding $\Delta F_{i0}/\Delta l_i$ and equality (9), the radial component of the total force f_i, acting upon the unit of length of the ith conductor with current in the plane of a thin plate, is found as:

$$f_i = \frac{\mu_0 I_i}{4\pi}\left\{\frac{I_i}{r_i}\left(\ln\frac{8r_i}{a} - \frac{3}{4}\right) + 2\sum_{j \neq i} I_j\left[\frac{1}{r_j + r_i}K(\lambda_{ij}) + \frac{1}{r_j - r_i}E(\lambda_{ij})\right]\right\}. \quad (21)$$

Substituting (21) into (10), we obtain the surface density of forces acting upon the plate from the side of a set of thin concentric circular conductors, with allowance made for the intrinsic magnetic field's effect on a conductor. The radial component of the stress tensor is determined by formulas (20) and (18), in which constants k_i and p_i are expressed in terms of quantities g_i, λ_{ij} by the formulas:

$$g_i = \frac{\mu_0(1-\sigma^2)}{4\pi E h} I_i\left\{\frac{I_i}{r_i}\left(\ln\frac{8r_i}{a} - \frac{3}{4}\right) + 2\sum_{j \neq i} I_j\left[\frac{1}{r_j + r_i}K(\lambda_{ij}) + \frac{1}{r_j - r_i}E(\lambda_{ij})\right]\right\}, \quad (22)$$

$$\lambda_{ij} = \frac{4r_i r_j}{(r_i + r_j)^2}.$$

The important characteristic of a system of circular conductors with current is their total magnetic moment. The interaction of this moment with the geomagnetic field can cause turning of a spacecraft with a film structure in the form of a solar battery. This results in a disturbance of its orientation and in reduction of the solar radiation flux onto the battery. The magnetic moment M of the system of conductors under consideration equals:

$$M = \pi\sum_{i=1}^{N} I_i r_i^2. \quad (23)$$

The problem of choosing the optimum parameters of a system of currents, providing deployment and stabilization of film structures by means of the Ampere forces can be formulated as a nonlinear optimization problem. It is necessary to select the current system's parameters $\{r_i, I_i\}$ in such a manner that stretching stresses in a film structure be maximum and its magnetic moment be minimum or equal to zero. Such a problem can be solved by selecting the magnitude and direction of currents in the film structure's conductors. As an example, we consider three circular conductors connected between themselves and located on a thin plate. We choose the values of their radii to be: $r_1 = 0.4$ m, $r_2 = 0.8$ m, $r_3 = R = 1.0$ m. We suppose the values of currents in the first and second conductors to be equal to $I_1 = 4$ A and $I_2 = 8$ A. The current in the third conductor is determined from the condition of zeroing total magnetic moment (23) of the system:

$$I_3 = -\frac{I_1 r_1^2 + I_2 r_2^2}{R^2} = -5.76 \text{ A}.$$

Figure 3 presents the plot of the radial component of the stress tensor in a thin plate with three concentric circular conductors with currents, whose values are given above, as a function of radius. The calculation was carried out by formulas (18), (20), and (22). The following parameters were chosen for calculation: $i = 3$; $h = 10^{-6}$ m; $E = 3 \cdot 10^9$ N/m^2; and $\sigma = 0.3$. For comparison, the plot shows also the value of a stress in the same plate, which arises under an effect of current I_3 flowing in a conductor located on the plate's edge. The calculation was carried out by formula (8). As follows from Fig. 3, in the presence of three conductors the stress in a plate depends on radius and bears discontinuity at connection points of conductors with a plate. The value of this stress considerably exceeds the stress generated by a single conductor. This example indicates that the use of several conductors as a stabilizing system allows one to essentially increase the value of a stress stretching the plate. Simultaneously, this scheme makes it possible to minimize the magnetic moment of a system by using differently directed currents of specific values.

Let us compare the efficiency of the considered technique of stabilizing film structures in space with the method based on using the centrifugal forces. For this purpose we estimate the value of a radial stress arising in a plate that rotates at angular frequency ω around the axis z. In this case the volume force is the centripetal force: $P_r = \rho h\omega^2 r$, where ρ is the density of the plate's material. For the rotating plate equation (11) has the form:

$$\frac{d}{dr}\left[\frac{1}{r}\frac{d}{dr}(ru_r)\right] = -\frac{1-\sigma^2}{E}\rho\omega^2 r.$$

The boundary condition is the absence of stress at the edge of a plate $\sigma_{rr}(r = R) = 0$. The solution of this equation with the given boundary condition allows one to find the radial stress:

$$\sigma_{rr} = \frac{\rho\omega^2 R^2(3+\sigma)}{8}\left(1 - \frac{r^2}{R^2}\right).$$

Now let us estimate the value of a radial stress at the plate's center $r = 0$. Letting: $\rho = 10^3$ kg/m^3 $R = 1.0$ m, $\sigma = 0.3$, $\omega = 2\pi/T$, $T = 15$ s, we obtain $\sigma_{rr} = 64$ N/m^2. Thus, the values of radial stresses, achieved with using both techniques, are comparable in the order of magnitude (see Fig. 3). The radial stress caused by inertial forces decreases down to zero at the plate's edge, while the stress caused by the Ampere force is nonzero everywhere. Therefore, there always exists the a region of the plate with the radius exceeding some specific value, in which the Ampere force produces stresses larger than the inertial force.

CONCLUSIONS

The magnetic field of a circular closed current results in appearance of the force acting upon a conductor with current generating this field. If the conductor is rigidly connected with the film it bounds, then the stresses and strains, caused by this force, arise in the film. It is shown that the stress does not depend on radius, and the strain grows towards the film's edge. The stress value considerably increases in the case, when several concentric conductors with current are located on the film. Each conductor is affected by its intrinsic magnetic field, as well as by the field generated by all remaining circular currents. It is shown that inside the circular conductor with the smallest radius the component of the stress tensor does not depend on radius. This component bears discontinuity at connection points of conductors with the film. And between two nearest concentric conductors it grows depending on radius. The choice of relationships between the values of radii and currents allows one to minimize the magnetic moment of the system of currents. The performed calculations have demonstrated the possibility of using the Ampere force for stabilization and deployment of thin film structures such as solar batteries.

ACKNOWLEDGMENTS

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APPENDIX

The self-induction coefficient of a closed conductor is determined by the expression:

$$L = \frac{\mu_0}{4\pi}\oint\oint\frac{d\mathbf{I}d\mathbf{I}'}{R_j}. \quad \text{(A1)}$$

In its calculation it is necessary to take into account the thickness of a conductor. If the conductor is supposed to be infinitely thin, then integral (A1) will logarithmically diverge at $R_j \longrightarrow 0$. This is due to the fact that both integrals are taken over the same contour. In order to estimate integral (A1) one should represent the self-induction coefficient as the sum $L = L_i + L_e$, where L_i and L_e are related with the magnetic field energy inside and outside the conductor, respectively [2]. The magnetic field inside the conductor can be supposed to coincide with the field of an infinite right cylinder $H = Ir/2\pi a^2$, where r is the distance from the cylinder's axis. This makes it possible to determine the internal part of the self-induction coefficient:

$$L_i = \frac{2}{I^2}\frac{\mu_0}{2}\int H^2 dV = \frac{\mu_0}{4}R. \quad \text{(A2)}$$

For estimating L_e one should have in mind, that the field outside the thin wire does not depend on the current distribution over its cross section. The energy of the external magnetic field do not change, if the current is assumed to flow over the wire surface only. To a sufficient degree of accuracy one can suppose [2], that:

$$L_e = \frac{\mu_0}{4\pi}\int_{R_j > a/2}\int\frac{d\mathbf{I}d\mathbf{I}'}{R_j}.$$

The integration is performed over all pairs of contour's points, the distance between which exceeds $a/2$. The integrand expression depends only on the central angle φ on which the chord R_j of the ring's circle leans upon (see Fig. 4). Substituting $R_j = 2R\,\sin(\varphi/2)$; $d\mathbf{I}d\mathbf{I}' = dldl'\cos\varphi$, we obtain:

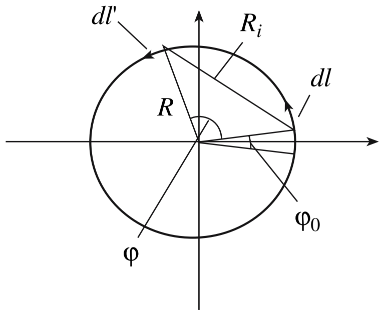

Fig. 4. The scheme of the integration contour for calculating the self-induction coefficient of a circular conductor.

$$L_e = \frac{\mu_0}{4\pi}\int_0^{2\pi} R d\varphi' 2\int_{\varphi_0}^{\pi} \frac{\cos\varphi R d\varphi}{2R\sin(\varphi/2)}$$
$$= \mu_0 R\left[-\ln\tan\frac{\varphi_0}{4} - 2\cos\frac{\varphi_0}{2}\right].$$

The lower limit of integration is determined from the condition $2R\sin(\varphi_0/2) = a/2$. Substituting $\varphi_0 \approx a/2R \ll 1$, we get:

$$L_e = \mu_0 R\left(\ln\frac{8R}{a} - 2\right). \quad \text{(A3)}$$

Adding (A2) and (A3), we obtain the estimate of integral (A1):

$$L = \mu_0 R\left(\ln\frac{8R}{a} - \frac{7}{4}\right). \quad \text{(A4)}$$

REFERENCES

1. *Handbook of Mathematical Functions*, Abramowitz, M. and Stegun, I.A., Eds., New York: Dover, 1965. Translated under the title *Spravochnik po spetsial'nym funktsiyam*, Moscow: Nauka, 1979.
2. Landau, L.D. and Lifshitz, E.M., *Elektrodinamika sploshnykh sred* (Electrodynamics of Continuous Media), Moscow: Nauka, 1982.
3. Landau, L.D. and Lifshits, E.M., *Teoriya uprugosti* (Theory of Elasticity), Moscow: Nauka, 1987.
4. Beernink, K., Pietka, G., Noch, J., *et al.*, Ultralight Amorphous Silicon Alloy Photovoltaic Modules for Space Application, *Paper V.6.2 Presented at the Materials Research Spring Meeting in San Francisco, CA, April 4, 2002.*
5. Guha, S., Yang, J., and Benerjee, A., Amorphous Silicon Alloy Solar Cells for Space Application, in *Proc. of the 2nd World Conference on Photovoltaic Solar Energy Conversion*, Vienna, Austria, 1998.
6. Melnikov, V.M. and Koshelev, V.A., *Large Space Structures Formed by Centrifugal Forces*, Amsterdam: Gordon and Breach Science Publ.,1999.