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Ionospheric generation mechanism of geomagnetic pulsations observed on the Earth's surface before earthquake

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Abstract

New generation mechanism of ULF geomagnetic oscillations observed on the Earth's surface in seismic zones is presented. This mechanism is based on the formation of periodic structure of ionospheric conductivity due to acoustic-gravity wave instability stimulated by DC electric field enhancement in the ionosphere. Interaction of the background electromagnetic ULF waves with such structure leads to an excitation of polarization currents and generation of narrow band gyrotropic waves at the 0.1–10 Hz frequency range in the ionosphere. The magnetic field of these waves can be observed on the ground. Since the growth of seismic activity is often accompanied by DC electric field enhancement on the ground and in the ionosphere, the suggested mechanism can be considered as a possible source of the seismogenic geomagnetic pulsations generated before and during earthquakes.

Keywords: Ionosphere; Irregularities; Gyrotropic waves; Earthquakes

1. Introduction

According to numerous publications, an enhancement of the geomagnetic oscillation intensity at the frequencies 0.01–10 Hz is often observed in the near epicenter zones of strong earthquakes before the main shock (Fraser-Smith et al., 1990; Kopytenko et al., 1994; Hayakawa et al., 1996). Practically, all the mechanisms suggested for explanation of this phenomenon were connected with different radiation sources situated within the lithosphere (Draganov et al., 1991; Fenoglio et al., 1994; Johnston et al., 1994; Molchanov and Hayakawa, 1995). Besides, Alperovich and Zheludev (1999) found the geomagnetic pulsations associated with ionospheric sources above the earthquake preparation zone.

In this paper, new generation mechanism of the seismogenic geomagnetic pulsations connected with the radiation source in the ionosphere is presented. This mechanism is based on the excitation of gyrotropic waves (GW) in the presence of periodic horizontal inhomogeneities of electric conductivity in the lower ionosphere. These waves first reported by Sorokin and Fedorovich (1982) propagate within thin layer of lower ionosphere along the Earth's surface in low and middle latitudes with small attenuation and with phase velocities, tens to hundreds of km/s. Some geophysical effects of GW are analyzed by Sorokin (1986, 1988) and Sorokin and Yaschenko (1988).

Chmyrev et al. (1989) found the seismic-related DC electric field disturbances in the ionosphere over the earthquake zone. The data of electric field enhancement up to 20 mV/m that occurred in the ionosphere are presented in Isaev et al. (2000). Discussion about possible mechanism of the observable electric field enhancement in the ionosphere was carried out by Sorokin et al. (2001). It has been shown that an enhancement of the DC electric field at definite conditions is accompanied by the generation of periodic inhomogeneous structure of electric conductivity in the lower ionosphere and the formation of geomagnetic field, which aligned with plasma layers in the upper ionosphere with the

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characteristic transverse spatial scale ∼10 km (Chmyrev et al., 1999; Sorokin et al., 1998; Sorokin and Chmyrev, 1999). Such layers observed by satellite as the variations of plasma density were reported by Chmyrev et al. (1997). So a growth of DC electric field in the ionosphere before earthquake leads to the formation of the quasi-periodic horizontal inhomogeneities of ionospheric conductivity. Various sources generate the electromagnetic noise at ULF range. The most powerful are thunderstorms. Oscillating noise electric field forms the polarization currents on the inhomogeneities of conductivity in the ionosphere. These horizontal currents with the spatial scale ∼10–100 km are considered as a source of GW. Generation and propagation of these waves lead to the excitation of narrow band electromagnetic radiation on the ground at the ULF range.

According to the electrodynamic model of the seismic related lower atmosphere and the ionosphere coupling (Sorokin et al., 1999, 2001; Sorokin and Yaschenko, 1999), the main cause of the ionosphere modification processes is the injection of radioactive substances and charged aerosols into the atmosphere. This leads to:

1. the formation of an external electric current near ground layer and the electric field increase in the ionosphere;
2. acoustic-gravity waves instability and spatial modulation of the conductivity in the ionosphere E layer;
3. appearance of polarization electric fields, which propagate into the upper ionosphere and generates the plasma density variations and the field-aligned currents at these altitudes; and
4. generation of seismic related geomagnetic pulsations that must be observed on the ground in epicenter region.

Our paper is devoted to question 4. It addresses theoretically the generation of GW in the 0.1–10 Hz range and focusses on the reaction of the lower ionosphere and the creation of geomagnetic pulsations registered on the ground before the earthquake.

2. Ultra-low-frequency electromagnetic waves in the lower ionosphere

The electric field components E_α of monochromatic wave with frequency ω satisfy Maxwell's equation:

$$\frac{\partial}{\partial x_\alpha}\left(\frac{\partial E_\alpha}{\partial x_\beta}-\frac{\partial E_\beta}{\partial x_\alpha}\right)=\mathrm{i}\,\frac{4\pi\omega}{c^2}\,j_\beta. \tag{1}$$

The equations of quasi-hydrodynamics for electrons, ions and molecules in the frequency range 0.01–10 Hz (Ginzburg, 1967) give Ohm's law in the following form (Sorokin,1988):

$$j_\alpha=\frac{\omega_\mathrm{i}c^2}{4\pi u^2}\left\{\frac{\omega_\mathrm{e}}{v_\mathrm{e}}\,E''_\alpha+[G_1\delta_{\alpha\beta}+G_2\varepsilon_{\alpha\beta\gamma}n_\gamma]E^\perp_\beta\right\}, \tag{2}$$

where $\omega_\mathrm{e},\omega_\mathrm{i}$ are the gyrofrequencies of electrons and ions, v_{ab} is the collision rate of a and b type particles, $v_\mathrm{e}=v_\mathrm{ei}+v_\mathrm{en}$, n_γ is the unit vector directed along the geomagnetic field, $u=B(4\pi MN)^{-1/2}$ is the Alfven velocity, $G_1=g(1+g^2)^{-1}$, $G_2=(1+g^2)^{-1}$, $g=v_\mathrm{e}/\omega_\mathrm{e}+\omega_\mathrm{i}/(v_\mathrm{in}-\mathrm{i}\omega)$, $\varepsilon_{\alpha\beta\gamma}$ is completely antisymmetric unit tensor, $\delta_{\alpha\beta}$ is the unit tensor, each of the tensor indexes notes the coordinates x, y and z. A product of the same index tensors denotes a sum of their components over this index (for example, $\varepsilon_{\alpha\beta\gamma}n_\gamma=\varepsilon_{\alpha\beta x}n_x+\varepsilon_{\alpha\beta y}n_y+\varepsilon_{\alpha\beta z}n_z$).

The first term in right-hand side of Eq. (2) corresponds to the current caused by the magnetic field-aligned electric field, and the second one—to transverse electric field. Since $v_\mathrm{e}/\omega_\mathrm{e}$ exceeds G_1 and G_2, more than four orders we neglect the parallel electric field in the wave. To obtain the equation for transverse components of the wave electric field let us substitute Eq. (2) into Eq. (1) and multiply the equation by the tensor $\varepsilon_{\alpha\beta\gamma}n_\beta$. As a result we find

$$\varepsilon_{\alpha\beta\gamma}n_\beta\,\frac{\partial}{\partial x_\mu}\left(\frac{\partial E^\perp_\mu}{\partial x_\gamma}-\frac{\partial E^\perp_\gamma}{\partial x_\mu}\right)-\mathrm{i}\,\frac{\omega\omega_\mathrm{i}}{u^2}(G_1\varepsilon_{\alpha\mu v}n_\mu+\delta_{\alpha v}G_2)E^\perp_v=0,$$

$$E^\perp_\alpha=(\delta_{\alpha\beta}-n_\alpha n_\beta)E_\beta. \tag{3}$$

Eq. (3) describes the propagation of the transverse low-frequency electromagnetic waves in weakly ionized ionospheric plasma with the ideal longitudinal conductivity. We assume that homogeneous external magnetic field is directed along the x-axis, the wave vector $\boldsymbol{k}$ lies in xy-plane, and φ is the angle between $\boldsymbol{k}$ and x. System (3) gives the dispersion equation for complex refraction index

$$(n+\mathrm{i}\kappa)^2=c^2\omega^2/k^2=\frac{(c^2\omega_\mathrm{i}/u^2\omega)[\mathrm{i}g(1+\cos^2\varphi)\pm(4\cos^2\varphi-g^2\sin^4\varphi)^{1/2}]}{2(1+g^2)\cos^2\varphi}. \tag{4}$$

In the case of a lower sign in Eq. (4), it determines the characteristics of the fast magnetosonic wave propagating with phase velocity $v=u$. This wave dissipates in the lower ionosphere. Let us consider the wave properties at the Alfven branch corresponding to upper sign in Eq. (4). Fig. 1 shows the plots for altitude dependence of the wave phase velocity and the λ/δ ratio, where λ is the wavelength and δ is a depth of the wave penetration calculated according to Eq. (4). In the upper ionosphere, where $v_\mathrm{in}\ll\omega_\mathrm{i}$, we obtain the following:

$$v=u\sqrt{\frac{2\omega}{\omega+\sqrt{\omega^2+v_\mathrm{in}^2}}}\cos\varphi,$$

$$\lambda/2\pi\delta=\left(\sqrt{\omega^2+v_\mathrm{in}^2}-\omega\right)\Big/v_\mathrm{in}.$$

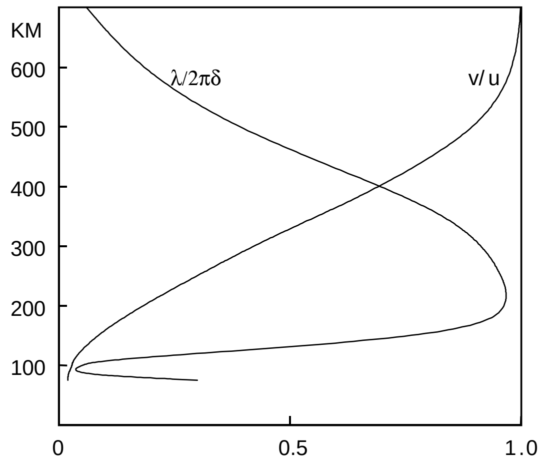

Fig. 1. Altitude dependence of the relative wave phase velocity v/u and relative dissipation $\lambda/2\pi\delta$. (u—Alfven velocity, λ—wavelength, and δ—depth of the wave penetration).

Note, that Alfven wave propagates with weak dissipation in the F2-layer, where $\omega \gg v_{\rm in}$, $v = u\cos\varphi$, $\lambda/2\pi\delta = v_{\rm in}/2\omega \ll 1$. At the lower altitudes, where $\omega \ll v_{\rm in}$, $v = u(2\omega/v_{\rm in})^{1/2}\cos\varphi$, $\lambda/2\pi\delta = 1$, the phase velocity decreases and dissipation grows. In E-region of the ionosphere, where $\omega_{\rm i} \ll v_{\rm in}$, we find that

$$v = 2u\cos\varphi$$

$$\sqrt{\frac{\omega(1+g^2)}{\omega_{\rm i}(2\cos\varphi\sqrt{1+g^2}+\sqrt{4\cos^2\varphi - g^2\sin^4\varphi})}},$$

$$\frac{\lambda}{2\pi\delta} = \frac{g(1+\cos^2\varphi)}{2\cos\varphi\sqrt{1+g^2}+\sqrt{4\cos^2\varphi - g^2\sin^4\varphi}},$$

$$g = \frac{v_{\rm e}}{\omega_{\rm e}} + \frac{\omega_{\rm i}}{v_{\rm in}}.$$

In the case of $g < 1$, the wave phase velocity is smaller than that of Alfven wave, and even their absorption is small. These waves propagate in the lower ionosphere with small attenuation and they have been first investigated by Sorokin and Fedorovich (1982) and called as gyrotropic waves (GW). They exist in weakly ionized plasma with magnetized electrons and non-magnetized ions. It is shown that GW propagate with small absorption along the Earth surface in the thin layer of the lower ionosphere at the heights 100–120 km. The wave properties in homogeneous and non-homogeneous environments are essentially different. Below, we analyze the dispersion characteristics of GW with the wavelengths considerably exceeding the width of the layer. In the lower ionosphere $\omega \ll v_{\rm in}$, the quantities G_1 and G_2 are expressed through Pedersen ($\sigma_{\rm P}$) and Hall ($\sigma_{\rm H}$) conductivities, i.e.

$$G_1 = 4\pi u^2\sigma_{\rm P}/c^2\omega_{\rm i}, \quad G_2 = 4\pi u^2\sigma_{\rm H}/c^2\omega_{\rm i}.$$

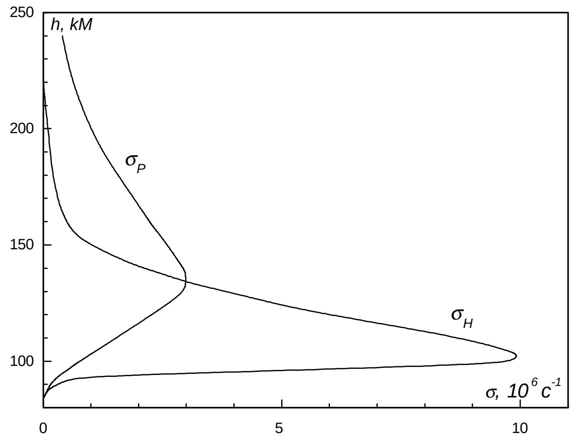

Fig. 2. Altitude distribution of the ionosphere Pedersen ($\sigma_{\rm P}$) and Hall ($\sigma_{\rm H}$) conductivities.

Taking into account this notion we can rewrite Eq. (1) in the following vector form:

$$\mathrm{rotrot}\,\mathbf{E} - \mathrm{i}\omega\frac{4\pi}{c^2}\widehat{\sigma\mathbf{E}} = 0, \tag{5}$$

where $\hat\sigma$ is the tensor of ionosphere conductivity.

3. GW propagation in the horizontal homogeneous ionosphere

For example, let us analyze the basic GW properties in the ionosphere considering their propagation along the horizontal magnetic field directed along x-axis in the Cartesian coordinates, here z-axis is directed upwards. Therefore, we obtain from Eq. (5) the equation for E_y and E_z components:

$$\left(\frac{\partial^2}{\partial x^2}+\frac{\partial^2}{\partial z^2}\right)\left(\mathrm{i}\frac{4\pi\omega\sigma_{\rm P}}{c^2}+\frac{\partial^2}{\partial x^2}\right)E_y + \mathrm{i}\frac{4\pi\omega\sigma_{\rm P}}{c^2}\frac{\partial^2 E_y}{\partial x^2}$$
$$= \left(\frac{4\pi\omega}{c^2}\right)^2(\sigma_{\rm H}^2+\sigma_{\rm P}^2)E_y,$$

$$\left(\mathrm{i}\frac{4\pi\omega\sigma_{\rm P}}{c^2}+\frac{\partial^2}{\partial x^2}\right)E_z = \mathrm{i}\frac{4\pi\omega\sigma_{\rm H}}{c^2}E_y. \tag{6}$$

The characteristic altitude distribution of the ionosphere conductivity, as shown in Fig. 2 displays two maximums, which correspond to two layers. At the upper layer, the value $\sigma_{\rm P}$ is greater than $\sigma_{\rm H}$. At lower layer (the Hall one), the value $\sigma_{\rm H}$ exceeds $\sigma_{\rm P}$ by two orders approximately. Therefore, we can assume that in Eq. (6) $\sigma_{\rm P} = 0$, for Hall layer. Assuming a dependence of unknown functions on x as $\exp(\mathrm{i}\omega x)$, we obtain

$$\frac{\mathrm{d}^2E_y}{\mathrm{d}z^2} - \left[k^2 - \left(\frac{4\pi\omega}{c^2k}\right)^2\sigma_{\rm H}^2(z)\right]E_y = 0,$$
$$E_z = -\frac{\mathrm{i}4\pi\omega\sigma_{\rm H}(z)}{c^2k^2}E_y. \tag{7}$$

Let us choose the origin of coordinates at a height point of the maximum of Hall conductivity. Interpolate the altitude distribution of Hall conductivity by function $\sigma_H(z) = \sigma_{H0}/\cosh(z/l)$ and introduce according to Appendix A new independent function $w = E_y[\cosh(z/l)]^s$ in Eq. (7). We find that

$$w = F\left(\frac{-s+kl}{2}, \frac{-s-kl}{2}, \frac{1}{2}, -\xi\right). \tag{8}$$

Since the field magnitude $E_y = E_0 w(1+\xi)^{s/2}$ tends to zero at $\xi \to \infty$, the quantity $(-s+kl)/2$ should be the integer negative number (including zero). It determines a spectrum of normal waves in the Hall layer:

$$S_n - k_n l = 2n, \quad n = 0, 1, 2 \ldots .$$

Substituting s from Eq. (A.1), we obtain the following:

$$\omega = a k_n[(k_n l + 2n)^2 + k_n l + 2n]^{1/2}, \quad a = c^2/4\pi\sigma_{H0} l. \tag{9}$$

Relation (9) gives the connection between frequency and wave number for each mode of GW. It follows from Eq. (9) that the phase velocity of GW is determined by the Hall conductivity. These waves exist in an interval of heights, where the Hall conductivity exceeds the Pedersen conductivity, and the weakly ionized plasma is essentially gyrotropical. The phase velocity v_n of normal waves is determined as

$$v_n = a[(k_n l + 2n)^2 + k_n l + 2n]^{1/2}.$$

Phase velocity of the basic mode $n = 0$ approaches zero with $\omega \to 0$, while the velocities of waves with $n = 1, 2 \ldots$, remain finite. In a high-frequency limit $\omega l/a \gg 1$, the basic mode $(n = 0)$ has dispersion $v_0 = (a\omega l)^{1/2}$. This corresponds to the transition to the homogeneous media. In the opposite case $\omega l/a \ll 1$, we have $v_0 = (a^2\omega l)^{1/3}$. In a low-frequency limit of the basic mode, the frequency dependence of the phase velocity coincides with the dispersion law of the waves propagated in an infinitely thin layer with Hall conductivity. The group velocity V_n is expressed through the phase by

$$V_n = 2v_n + a^2[1 - (4n+1)(1 + 4v_n^2/a^2)^{1/2}]/4v_n.$$

For a wave with $n = 0$ at $\omega l/a \ll 1$, we have $V_0 = (3/2)v_0$, i.e. the group velocity exceeds that of the phase. Let us make the numerical estimations of the parameters. Assuming the maximal value of Hall conductivity in the layer $\sigma_{H0} = 6 \times 10^6 c^{-1}$ and their semi-thickness $l = 15$ km, we obtain $a = 100$ km/s. For example, the waves with the period $T = 60$ s $(\omega l/a = 1.5 \times 10^{-2})$ have phase speeds: $v_0 = 0.24a = 24$ km/s, $v_1 = 2.5a = 250$ km/s, $v_2 = 450$ km/s. It is necessary to notice that the considered layer differs in properties from wave guide, as Eq. (7) to which a field satisfies is not the wave equation. Nevertheless, the waves within the layer propagate without attenuation, but with frequency dispersion, and the modes with large numbers have the large phase velocity.

The vertical distribution of normal wave amplitudes is determined by formula (8). In Fig. 3 the dependence of E_{yn} on z for three normal waves are given. It is seen from Fig. 3 that within the conducted layer the field amplitude of the basic mode of the wave is practically homogeneous. The field of the waves with higher numbers significantly changes within the layer. Outside the layer, the field exponentially increases with distance over the wavelength of the appropriate mode. For the basic mode at $n = 0$, we have

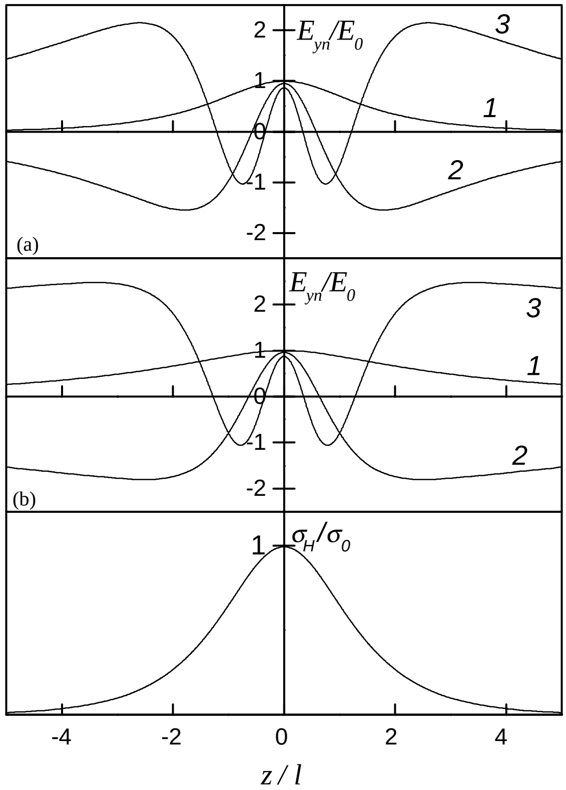

Fig. 3. Altitude dependence of the relative electric field component E_{yn}/E_0 for three wave modes. The bottom panel shows the model of the Hall conductivity altitude distribution: (a) $\omega l/a = 1.0$; (b) $\omega l/a = 0.2$; (1) $n = 0$; (2) $n = 1$; (3) $n = 2$.

$$E_{y0} = E_0[\cosh(z/l)]^{-(\omega l/a)^{2/3}}. \tag{10}$$

The slow change of the basic mode field across the conducting layer allows to apply the approached method for its accounts. This method consists of the replacement of real distribution of conductivity by infinitely thin conducting layer $\sigma_H^2(z) = \sigma_{H0}^2\delta(z/2l)$ under the condition $kl \ll 1$ (where $\delta(z/2l)$ is Dirac delta-function, and l is the characteristic thickness of a conducting layer). Using this method, we shall estimate the accuracy of the approached solution, comparing it with exact one (10). Let us integrate Eq. (7) on z and tend l to zero, then we obtain

$$l\left\{\frac{dE_y}{dz}\right\} + 2\left(\frac{\omega}{ak}\right)^2 E_y(0) = 0; \quad \{E_y\} = 0,$$

where brackets $\{\dots\}$ designate the difference of quantities above and below the conducting layer. The electric field is determined from Laplace equation outside this layer. Its solution is

$$z>0 \;\; E_y=E_0\exp(-kz),\quad z<0 \;\; E_y=E_0\exp(kz).$$

Substituting this solution in the boundary condition, we obtain dispersion relation for a wave in the layer $k^3=\omega^2/la^2$. Taking into account this relation we shall write down the electric field distribution in the upper semi-area:

$$z>0 \;\; E_y=E_0\exp(-kz)=E_0\exp\left\{-\frac{z}{l}\left(\frac{\omega l}{a}\right)^{2/3}\right\}.$$

Let us compare the obtained approached solution with the exact one, which follows from Exp. (10) at $z/l \gg 1$:

$$E_y=E_0 2^{(\omega l/a)^{2/3}}\exp\left\{-\frac{z}{l}\left(\frac{\omega l}{a}\right)^{2/3}\right\}.$$

Difference between these solutions is determined by the factor $2^{(\omega l/a)^{2/3}} \approx 1+(kl)\ln 2$, which is of the order of unit in the considered approximation $kl \ll 1$. Thus, the approached method with a sufficient accuracy allows to analyze the effects of the wave generation and propagation in the thin layers by introducing the appropriate boundary conditions.

Below, we take advantage of this method for the analysis of absorption of GW. We believe that the conductivity of ionosphere is concentrated within two flat layers near the plane $z=0$. We have non-zero Hall conductivity in the bottom layer and non-zero Pedersen conductivity in the top layer. The ideally conducting Earth coincides with the plane $z=-z_l$. In the Earth-ionosphere layer, the conductivity is assumed to be zero. From Eq. (6) we obtain the equations for E_y in the layer with Hall conductivity

$$\frac{\partial^2}{\partial x^2}\left(\frac{\partial^2}{\partial x^2}+\frac{\partial^2}{\partial z^2}\right)E_y-\left(\frac{4\pi\omega}{c^2}\right)^2\sigma_{\mathrm{H}}^2(z)E_y=0 \tag{11}$$

and in the layer with Pedersen conductivity

$$\left(\frac{\partial^2}{\partial x^2}+\frac{\partial^2}{\partial z^2}\right)E_y+\mathrm{i}\,\frac{4\pi\omega}{c^2}\,\sigma_{\mathrm{P}}(z)E_y=0. \tag{12}$$

Integration of the equations on z at the condition that the width of the layers will tend to zero leads to boundary conditions for components E_y and its vertical derivative at transition through each of these layers. Assuming the field dependence on x as $\exp(\mathrm{i}kx)$ we obtain a boundary condition for the layer with Hall conductivity

$$\left\{\frac{\mathrm{d}E_y}{\mathrm{d}z}\right\}+\left(\frac{4\pi\omega}{kc^2}\right)^2\int_{-\infty}^{\infty}\sigma_{\mathrm{H}}^2(z)\,\mathrm{d}z\,E_y,\quad \{E_y\}=0$$

and for a layer with Pedersen conductivity

$$\left\{\frac{\mathrm{d}E_y}{\mathrm{d}z}\right\}+\mathrm{i}\,\frac{4\pi\omega}{c^2}\int_{-\infty}^{\infty}\sigma_{\mathrm{P}}(z)\,\mathrm{d}z\,E_y=0,\quad \{E_y\}=0.$$

Equating the distance between the layers to zero and putting these expressions, we find the boundary conditions connecting mean values of tangential components of an electrical field and its vertical derivative above and below the ionosphere

$$\left\{\frac{\mathrm{d}E_y}{\mathrm{d}z}\right\}+\left[\left(\frac{4\pi\omega}{kc^2}\right)^2\int_{-\infty}^{\infty}\sigma_{\mathrm{H}}^2(z)\,\mathrm{d}z + \mathrm{i}\,\frac{4\pi\omega}{c^2}\int_{-\infty}^{\infty}\sigma_{\mathrm{P}}(z)\,\mathrm{d}z\right]E_y=0,\quad \{E_y\}=0. \tag{13}$$

As the velocity of electromagnetic wave in the upper ionosphere and in the Earth-ionosphere layer is much higher than the velocity of GW, the fields in these areas determined from the Laplace equation have the following form:

$$E_y=E_{y0}\exp(-kz),\quad z>0,$$

$$E_y=E_{y0}\sinh[k(z+z_1)]/\sinh(kz_1),\quad -z_1<z<0.$$

Substituting this solution in the boundary conditions (13), we obtain

$$\omega^2+\mathrm{i}v\omega k^2-k^3a^2l[1+\coth(kz_1)]=0, \tag{14}$$

where

$$a=\frac{c^2}{4\pi\sqrt{l\int \mathrm{d}z\sigma_{\mathrm{H}}^2(z)}}=\frac{c^2}{4\pi l\sigma_{\mathrm{H0}}},$$

$$v=\frac{c^2\int \mathrm{d}z\sigma_{\mathrm{P}}(z)}{4\pi\int \mathrm{d}z\sigma_{\mathrm{H}}^2(z)},\quad l=\frac{\int \mathrm{d}z\sigma_{\mathrm{H}}^2(z)}{\sigma_{\mathrm{H0}}^2}.$$

Expression (14) is the dispersion equation for GW in the lower ionosphere accounting absorption and influence of the ideally conducting Earth.

4. Calculation of ULF field oscillations on the ground during the occurrence of periodic conductivity inhomogeneities in the ionosphere

We will consider the generation of GW propagated along an x-axis by inhomogeneities of ionospheric conductivity. Let the coordinate dependence of conductivity have a form

$$\sigma_{\mathrm{H}}=\sigma_{\mathrm{H0}}(z)+\sigma_{\mathrm{H1}}(x,z);\quad \sigma_{\mathrm{P}}=\sigma_{\mathrm{P0}}(z)+\sigma_{\mathrm{P1}}(x,z),$$

where the index 0 designates the undisturbed conductivity, and index 1 is its disturbance. Let us present the electric field as a sum $E_y=E_{y0}+E_{y1}$, where E_0 is background electric and E_1 represents a field arising during the occurrence of ionospheric conductivity inhomogeneities. Let us substitute these expressions to equalities (11) and (12). Assuming that the conductivity disturbances are small, and taking into account the first order disturbances of the electric field we obtain the equations for E_{y1} within the layers with Hall conductivity:

$$\frac{\partial^2}{\partial x^2}\left(\frac{\partial^2}{\partial x^2}+\frac{\partial^2}{\partial z^2}\right)E_{y1}-\left(\frac{4\pi\omega}{c^2}\right)^2\sigma_{\mathrm{H0}}^2(z)E_{y1} = \left(\frac{4\pi\omega}{c^2}\right)^2\sigma_{\mathrm{H0}}^2(z)h(x)E_{y0}$$

and Pedersen conductivity

$$\left(\frac{\partial^2}{\partial x^2}+\frac{\partial^2}{\partial z^2}\right)E_{y1}+\mathrm{i}\,\frac{4\pi\omega}{c^2}\sigma_{\mathrm{P}}(z)E_{y1}$$
$$=-\,\mathrm{i}\,\frac{4\pi\omega}{c^2}\,\sigma_{\mathrm{P}0}(z)\,p(x)E_{y0}.$$

In these equations, the relative disturbances of a Hall and Pedersen conductivities are entered

$$h(x)=\frac{\sigma_{\mathrm{H}1}(x,z)}{\sigma_{\mathrm{H}0}(z)},\quad p(x)=\frac{\sigma_{\mathrm{P}1}(x,z)}{\sigma_{\mathrm{P}0}(z)}.$$

Let the unknown values depend on coordinate x as $\exp(\mathrm{i}kx)$. Using a method stated above, we shall obtain the boundary conditions connecting the tangential components of the electrical field and its vertical derivative above and below the ionosphere

$$\left\{\frac{\mathrm{d}E_{y1}}{\mathrm{d}z}\right\}+\frac{1}{la^2}\left[\left(\frac{\omega}{k}\right)^2+\mathrm{i}\omega v\right]E_{y1}$$
$$=-\,\frac{1}{la^2}\left[\left(\frac{\omega}{k}\right)^2 f_{\mathrm{H}}(k,0)+\mathrm{i}\,f_{\mathrm{P}}(k,0)\right],$$

$$\{E_{y1}\}=0. \tag{15}$$

In Eq. (15) the following designations are introduced:

$$f_{\mathrm{H}}(k,z)=\int_{-\infty}^{\infty}\mathrm{e}^{\mathrm{i}kx}h(x)E_{y0}(x,z,\omega)\,\mathrm{d}x,$$

$$f_{\mathrm{P}}(k,z)=\int_{-\infty}^{\infty}\mathrm{e}^{\mathrm{i}kx}\,p(x)E_{y0}(x,z,\omega)\,\mathrm{d}x.$$

During the performance of an inequality $(\omega/k)^2 \gg \mathrm{i}v\omega$ absorption of GW is weak. Let us assume that the relative disturbance of Hall conductivity is not less than the relative disturbance of Pedersen conductivity. This condition is not basic, however it allows one to simplify the calculations. Thus, the second term in the right-hand side of Eq. (15) can be neglected. In such a case, when horizontal scale of $E_0(x,z,\omega)$ variation, representing the undisturbed field, exceeds the horizontal scale of disturbed area, it can be taken from a sign of integral: $f_{\mathrm{H}}(k,z)=E_{y0}(z,\omega)H(k)$, where $H(k)$ is Fourier image of $h(x)$

$$H(k)=\int_{-\infty}^{\infty}\mathrm{d}x\,\mathrm{e}^{\mathrm{i}kx}h(x).$$

Substituting the solution for fields above the ionosphere and in the Earth-ionosphere layer into the boundary conditions (15), we obtain

$$E_{y1}(\omega,k)=-E_{y0}(\omega)\,\frac{\omega^2H(k)}{\omega^2-\omega_0^2(k)+\mathrm{i}v\omega k^2}, \tag{16}$$

where $\omega_0^2(k)=2la^2|k|^3$.

As an example, we choose the relative disturbance of Hall conductivity as $h(x)=C(x)\cos(k_0x)$, where $C(x)$ slowly varies on scale $l_0=2\pi/k_0$. Taking into account the quasi-periodic character of $h(x)$ variation in Eq. (16) it is possible to find the approximate expression for inverse Fourier transform in k, having obtained the spatial structure of a spectrum of GW

$$\frac{E_{y1}(\omega,x)}{E_{y0}(\omega)}=\frac{\mathrm{i}\omega^2}{4\omega_0u}\left[\mathrm{e}^{-\mathrm{i}k_0x}\int_0^{\infty}C(x+s)\mathrm{e}^{\mathrm{i}sq}\,\mathrm{d}s\right.$$
$$\left.+\,\mathrm{e}^{\mathrm{i}k_0x}\int_0^{\infty}C(x-s)\mathrm{e}^{\mathrm{i}sq}\,\mathrm{d}s\right], \tag{17}$$

where $u=\mathrm{d}\omega_0(k_0)/\mathrm{d}k_0,\ q=\omega^2-\omega_0^2+\mathrm{i}v\omega k_0^2/2\omega_0u$.

The electric field disturbance E_{y1} generates an additional current in the ionosphere. The magnetic field of this current can be observed on the Earth's surface. Using the Maxwell equation $\mathrm{rot}\,\mathbf{E}=\mathrm{i}\omega\mathbf{B}/c$, and the expression determining the electric field in the Earth-ionosphere layer, it is easy to show that the magnetic disturbance on the ground is also defined by Eq. (17):

$$\left.\frac{B_{x1}(x,\omega)}{B_{x0}(\omega)}\right|_{z=-z_1}=\left.\frac{E_{y1}(x,\omega)}{E_{y0}(\omega)}\right|_{z=0}.$$

Let us choose the model distribution $C(x)=A\exp(-|x|/L)$. Then integrals (17) are expressed in elementary functions

$$\frac{B_{x1}(x,\omega)}{B_{x0}(\omega)}=A\,\frac{(\omega/\Omega)^2}{1+(qL)^2}\,\{\mathrm{e}^{-|x|/L}[\sin(k_0|x|)$$
$$-(qL)\cos(k_0|x|)]+\mathrm{i}\exp[\mathrm{i}(k_0+q)|x|]\}, \tag{18}$$

where $\Omega=\sqrt{2\omega_0u/L}$.

The estimates show (Fatkullin et al., 1981) that $a=4\times10^4$ m/s, $v=2\times10^8$ m^2/s. According to the satellite data (Chmyrev et al., 1997), the characteristic sizes of the disturbed ionosphere area $2L$ is few hundred kilometers, and spatial scale of disturbance $d=l/2=\pi/k_0$ is tens of kilometers. For accounts we assume the size of the relative disturbance of Hall conductivity $A=0.1$ and $L=10^5$ m. The diagram of relative disturbance of the geomagnetic fluctuations spectrum $|B_{x1}(x,\omega)/B_{x0}(\omega)|$, found from formula (18) for epicenter, is given in Fig. 4. It is seen that the relative disturbance is maximal at the $\omega_{\mathrm{m}}\sim 2$ and 5 Hz frequencies for two spatial scales of disturbance $d=l/2=\pi/k_0$, and its magnitude in epicenter reaches approximately 20–25% of the undisturbed value. It has been calculated by formula (18) that the average value of the spectrum maximum amplitude $\langle B_1(x,\omega_m)\rangle/B_0(\omega_m)$ depends on the x distance from epicenter of the disturbed region. The result of this calculation presented in Fig. 5 enables one to find a spatial scale of the region, where ULF pulsations can be observed. As follows from Fig. 5, this spatial scale is of the order of 200–300 km.

5. Conclusion

It is shown that the interaction of background electromagnetic noise with periodic horizontal inhomogeneities

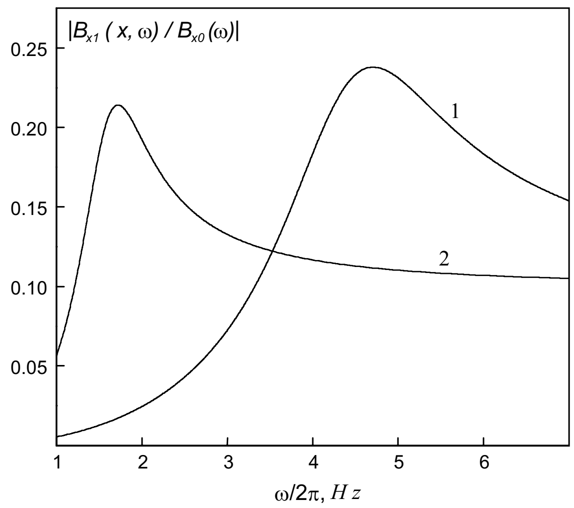

Fig. 4. The calculated normalized spectrum of geomagnetic oscillations for different spatial scales of inhomogeneities. (1) $d = 15$ km; (2) $d = 30$ km.

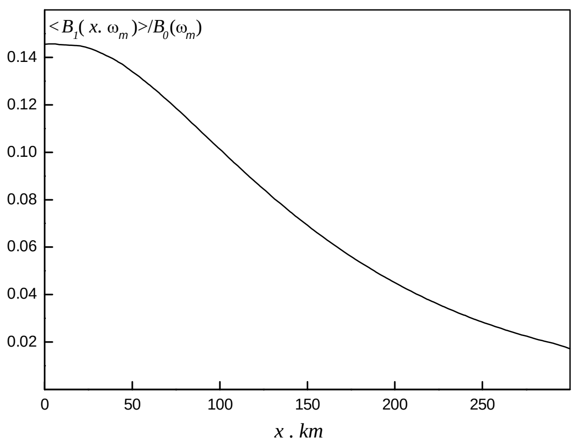

Fig. 5. Dependence of average value of the spectrum maximum amplitude on the x distance from the epicenter of the disturbed region.

of ionospheric conductivity with the spatial scale over 10 km results in the generation of electromagnetic waves with a narrow-band spectrum. Various sources generate background electromagnetic noise in ULF range. The most powerful are thunderstorms. Oscillating noise of the electric field forms the polarization currents by conductivity inhomogeneities in the ionosphere. These horizontal periodic electric currents with the spatial scale ∼10 km are considered as a source of GW. Generation and propagation of these waves lead to the excitation of the narrow band magnetic oscillations at 0.1–10 Hz frequencies on the ground, which result in interference effect. Its value in epicenter reaches approximately 20–25% above the undisturbed background level. The spectral maximum amplitude decreases dependence on a distance from epicenter. The spectral maximum frequency decreases monotonously the dependence on the spatial scale of inhomogeneities.

The calculations show that the characteristic frequency of geomagnetic pulsations connected with GW belongs to ULF range 0.1–10 Hz of electromagnetic emissions registered, for example, by Kopytenko et al. (2001) before earthquakes. The electromagnetic emissions with the maximal frequency over units of Hz during seismic activity and volcanic eruptions have been observed by Rauscher and Van Bise (1999). From the presented model, it follows that if some irregularities of the ionospheric conductivity with various spatial scales exist simultaneously, then perturbations on the Earth surface can be observed in several, connected to them by spectral bands. Note that the spectrum maximum frequency depends essentially on the volume of Hall and Pedersen ionospheric conductivity, angle of magnetic field inclination, azimuth of direction of the wave propagation, and spatial scale of conductivity irregularities. The variations of these parameters lead to the change of the spectrum maximum frequency over a wide range. Using a simple model, we illustrate the generation mechanism of geomagnetic pulsations. For analysis of the experimental data received during seismic activity on the basis of this mechanism, it is necessary to develop a submitted model. Therefore, in the present work, we are limited by the estimations of the geomagnetic pulsation characteristics. For discovering ULF pulsations connected with GW in the ionosphere, it is necessary to provide the precise measurements of the wave phase delay in some points of the Earth's surface. The horizontal velocities of the ULF waves propagating from the epicenter must be of the order of 10–100 km/s. Those velocities should be increased depending on the frequency growth.

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Appendix A

Below we find the solution of Eq. (7). For the function w we obtain

$$\frac{d^2 w}{dz^2} - \frac{2s}{l}\tanh\left(\frac{z}{l}\right)\frac{dw}{dz} + \left\{\frac{s(s+1)[\sinh(z/l)]^2}{l^2[\cosh(z/l)]^2} - \frac{s}{l^2} + \left(\frac{4\pi\omega\sigma_{H0}}{c^2 k\cosh(z/l)}\right)^2 - k^2\right\} w = 0.$$

Parameter s is determined from a condition of the constancy of coefficient in the bracket at unknown w

$$\frac{s(s+1)[\sinh(z/l)]^2 + (4\pi\sigma_{H0}\omega l/c^2k)^2}{l^2[\cosh(z/l)]^2} = \frac{s(s+1)}{l^2}.$$

Hence, we obtain

$$s = \frac{1}{2}\left\{-1 + \sqrt{1 + 4\left(\frac{4\pi\sigma_{H0}\omega l}{c^2k}\right)^2}\right\}. \quad \text{(A.1)}$$

The equation for w with parameter s determined from Eq. (A.1) has the following form:

$$\frac{d^2w}{dz^2} - \frac{2s}{l}\,\mathrm{th}\left(\frac{z}{l}\right)\frac{dw}{dz} + \left(\frac{s^2}{l^2} - k^2\right)w = 0.$$

The variable replacement according to $\xi = [\sinh(z/l)]^2$ transforms this equality to the hypergeometric equation (Bateman and Erdelyi, 1953)

$$\xi(\xi+1)w''_{\xi\xi} + \left[(1-s)\xi + \frac{1}{2}\right]w'_{\xi} + \frac{1}{4}(s^2 - k^2l^2)w = 0.$$

The solution of this equation is the hypergeometric function:

$$w = F\left(\frac{-s+kl}{2}, \frac{-s-kl}{2}, \frac{1}{2}; -\xi\right). \quad \text{(A.2)}$$

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