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Gyrotropic waves in the mid-latitude ionosphere

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Abstract

The propagation of electromagnetic ULF perturbations in the thin conductive ionospheric layer in the magnetic meridian plane is considered. The dispersion relation for the waves propagating in the horizontal direction is obtained accounting for the influence of the conjugate ionosphere. The dependence of the phase velocity and absorption coefficient as the function of the wave frequency and magnetic field inclination are found. It is shown that in the frequency range 0.001–1 Hz the phase velocity increases from a few units to a few tenths of km/s depending on the frequency and so far follows the approximate dependence $\omega^{1/3}$. The phase velocity decreases with the increase in the magnetic field inclination. The propagation of quasi-harmonic and unipolar impulses of gyrotropic waves in the horizontal direction is analyzed. The dependence of their characteristics on the values of conductivity tensor components and on the magnetic field inclination is evaluated. The generation of gyrotropic waves by the electromagnetic field of atmospheric origin in the presence of ionospheric inhomogeneities is considered. The calculation of the power spectra of the geomagnetic field oscillations at the ground level is carried out. The dependence of the spectrum on the magnetic field inclination is obtained. Some features of geophysical phenomena, associated with propagation of gyrotropic waves in the low ionosphere are discussed.

Keywords: Gyrotropic waves; Mid-latitude ionosphere; Electric fields in the ionosphere

1. Introduction

A long time ago Herron (1965) using the data of mid-latitude geophysical observatories pointed out that *Pi2* and *Pc3* geomagnetic pulsations can propagate along the Earth's surface with the phase velocity of the order of a few tenths of km/s. The peculiarities of spatio-temporal distribution of the *Pi2* pulsations at mid-latitudes have been extensively studied by Gokhberg et al. (1973). These authors have estimated the pulsation group velocity along the Earth's surface and found the value of the 40–300 km/s. Gogatishvili (1979) provided the analyses of the spatio-temporal distribution of *Pc5, Pc6* pulsations with periods 5–8 min. He also pointed out the phase shift that was of the order of 10–15 km/s. A possible influence of the ionosphere on the propagation of geomagnetic pulsations has been discussed by Piddington (1959). This author came to conclusion that strong atmospheric activity can serve as source for generation of specific ionospheric perturbations and geomagnetic pulsations propagating with the phase velocities of the same order. Analyses of magnetograms

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carried out by some researches (Sorokin and Fedorovich, 1982a; Alperovich et al., 1982; Alperovich and Zheludev, 1999; Ismaguilov et al., 2001) provided the evidence for the geomagnetic pulsation generation by earthquakes propagating from the epicenters along the Earth's surface with the velocities from a few units to a few tenths of km/s. The greater the signal period the smaller the velocities. Similar velocities of the ionospheric perturbations have been observed in the "MASSA" experiment (e.g., Alperovich et al., 1985) and during rocket launches (Karlov et al., 1984). Sorokin and Fedorovich (1982a, b) and Sorokin (1988) have attributed such ionospheric and geomagnetic perturbations to the so-called gyrotropic waves. These waves (with frequencies 0.001–1 Hz) can propagate in the conductive ionospheric layer with velocities 10–100 km/s possessing small damping and dispersion. Outside the conductive layer the wave electric field produces a strong electron motion and thus the waves are observed in the form of the ionospheric perturbations. Recently Sorokin et al. (2003) developed a model for the formation of the narrow band ULF spectrum near the Earth's surface by gyrotropic waves driven by thunderstorm activity. Similar effects can display themselves in seismoactive regions as the result of the increase in the electric field (Sorokin et al., 1998; Chmyrev et al., 1999) or due to acoustic-gravity waves generated by seismic sources (Mareev et al., 2002). The mechanism of formation of the geomagnetic pulsation spectra can be used for monitoring of seismic activity (Kopytenko et al., 2002; Hayakawa et al., 1996). All the above-mentioned geophysical phenomena are associated with generation and propagation of the gyrotropic waves which may be used for relevant geophysical interpretation.

The present paper represents the generalization of the theory of gyrotropic waves accounting for the effects of oblique external magnetic field and the effects due the influence of the conjugate ionosphere. The paper is organized in the following fashion: Section 2 is devoted to derivation of general dispersion relation for the gyrotropic waves. The spatio-temporal characteristics of these waves are analyzed in Section 3. In Section 4 the role of the magnetic field inclination in the formation of geomagnetic pulsation spectra is investigated. The Appendix gives the details of our calculations. Our discussion and conclusions are found in Section 5.

2. Dispersion and damping of gyrotropic waves in the mid-latitude ionosphere

In a uniform partly ionized plasma the phase velocity v_p of the gyrotropic waves and their damping are given by (cf. Sorokin et al., 2003)

$$v_p = 2u\cos\varphi\sqrt{\frac{\omega(1+g^2)}{\omega_i\left(2\cos\varphi\sqrt{1+g^2}+\sqrt{4\cos^2\varphi-g^2\sin^4\varphi}\right)}},$$
$$u = B/\sqrt{4\pi MN},$$
$$\varepsilon = \frac{\lambda}{2\pi\delta} = \frac{g(1+\cos^2\varphi)}{2\cos\varphi\sqrt{1+g^2}+\sqrt{4\cos^2\varphi-g^2\sin^4\varphi}},$$
$$g = \frac{\nu_e}{\omega_e}+\frac{\omega_i}{\nu_{in}},$$

where u stands for the Alfven velocity, M and N are the mass and the ion number density, respectively, ω_e and ω_i the electron and the ion gyrofrequencies, ν_e and ν_{in} the electron and the ion collision frequencies, λ the wavelength, δ the distance at which the field decreases in e-times, φ the angle between the direction of the wave propagation and the external magnetic field **B**. This expression is valid when $\omega \ll \omega_i N/N_m$, where N_m is the neutral particle number density. In the E-layer $g<1$ and thus the gyrotropic wave phase velocity is much smaller than the Alfven velocity. Moreover, the waves possess a small damping. Contrary to the Alfven waves, the phase velocity of these waves strongly depends on the collision frequencies. These specific waves are localized in the region where the electrons are magnetized whereas the ions not. The gyrotropic waves can be described in terms of the conductivity tensor of partly ionized plasma (Sorokin et al., 2003). They possess small damping if the off-diagonal elements of the dielectric tensor are greater than the diagonal terms. For the sake of simplicity we obtain the dispersion relation for these waves in a limiting case of a thin conductive E-layer when the wavelength is greater than the depth of the layer.

The electric **E** and magnetic field **b** perturbations can be found from Maxwell equations

$$\nabla\times\mathbf{E} = -\frac{1}{c}\frac{\partial\mathbf{b}}{\partial t}, \quad \nabla\times\mathbf{b} = \frac{4\pi}{c}\mathbf{j}+\frac{1}{c}\frac{\partial\mathbf{E}}{\partial t},$$
$$\nabla\cdot\mathbf{E} = 4\pi q, \quad \nabla\cdot\mathbf{b} = 0, \tag{1}$$

where q is the electric charge density, **j** is the electric current. Eqs. (1) should be supplemented by the generalized Ohm's law given by

$$\mathbf{j} = \sigma_{11}\frac{\mathbf{B}(\mathbf{E}\cdot\mathbf{B})}{B^2}+\sigma_P\left\{\mathbf{E}-\frac{\mathbf{B}(\mathbf{E}\cdot\mathbf{B})}{B^2}\right\}+\sigma_H\frac{\mathbf{B}\times\mathbf{E}}{B}. \tag{2}$$

Here σ_{11} stands for the field-aligned conductivity, σ_P and σ_H are the Pedersen and Hall conductivities. Furthermore, we introduce a local Cartezian system of coordinates (x,y,z) with the longitudinal x and latitudinal y coordinates. The z-axis of our system coincides with local vertical direction. A uniform external magnetic field **B** lies in the (x,z) plane and directed under the angle φ to the x-axis. The ionosphere is assumed to be plane with the electric conductivity that might depend on the z-coordinate.

Let us consider the propagation of the electromagnetic field in the (x,z) plane assuming $\partial/\partial y = 0$. Due to the high mobility of the electrons along the external magnetic field $\sigma_{11} \ll \sigma_P, \sigma_H$. Thus, setting in Eq. (2) $\sigma_{11} \to \infty$ we find $(\mathbf{E}\cdot\mathbf{B}) = 0$. The latter gives

$$E_x = -E_z \tan\varphi \tag{3}$$

From Eqs. (1) and (2) one finds the equation for the perpendicular components of the electric field in the conductive ionosphere

$$\left(\Delta - \frac{1}{c^2}\frac{\partial^2}{\partial t^2}\right)(\mathbf{B}\times\mathbf{E}) - \mathbf{B}\times\nabla(\nabla\cdot\mathbf{E}) = \frac{4\pi}{c^2}\left\{\sigma_P\left(\mathbf{B}\times\frac{\partial\mathbf{E}}{\partial t}\right) - \sigma_H B\frac{\partial\mathbf{E}}{\partial t}\right\}. \tag{4}$$

With the help of Eqs. (4) and (3) one finds the system of equations for the electric field components

$$\left(\frac{\partial^2}{\partial x^2} + \frac{\partial^2}{\partial z^2} - \frac{\partial^2}{c^2\partial t^2}\right)E_y = \frac{4\pi}{c^2}\left(\sigma_P\frac{\partial E_y}{\partial t} - \frac{\sigma_H}{\cos\varphi}\frac{\partial E_z}{\partial t}\right)$$
$$\left[\left(\sin\varphi\frac{\partial}{\partial z} + \cos\varphi\frac{\partial}{\partial x}\right)^2 - \frac{\partial^2}{c^2\partial t^2}\right]E_z = \frac{4\pi}{c^2}\left(\sigma_P\frac{\partial E_z}{\partial t} + \sigma_H\cos\varphi\frac{\partial E_y}{\partial t}\right). \tag{5}$$

The altitude distribution of σ_P and σ_H is depicted in Fig. 1. Since the wavelength is much greater than the typical scale of the conductivity inhomogeneity one can replace these equations by the relevant boundary conditions at the E-layer and obtain the solution of the system (5). Outside the layer we set the conductivities to zero. Let the thin E-layer to coincide with the $z = 0$ plane. Assuming all perturbed values to vary as $\exp(ikx - i\omega t)$, we write the boundary conditions for the components of the electric field at the conductive thin ionosphere in the form (see Appendix, Eq. (A.5))

$$\left\{\frac{dE_y}{dz}\right\}_{z=0} + \frac{\omega^2 + i\nu\omega k^2\cos^2\varphi}{k^2a^2l\cos^2\varphi}E_y(0) = -\frac{2\omega\sin\varphi}{ka\cos^2\varphi}\frac{dE_z(z=-0)}{dz}, \quad \{E_y\}_{z=0} = 0;$$
$$\left\{\frac{dE_z}{dz}\right\}_{z=0} = -i\frac{\nu\omega}{a^2l\sin^2\varphi}E_z(z=+0),$$
$$\{E_z\}_{z=0} = -\frac{\omega}{2ka\sin\varphi}E_y(0). \tag{6}$$

Here the following abbreviations are

$$a = c^2\Big/4\pi\sqrt{l\int_{-\infty}^{\infty}dz\sigma_H^2(z)}, \quad l\sigma_H^2(0) = \int_{-\infty}^{\infty}dz\sigma_H^2(z),$$
$$\nu = a^2l/a_P. \tag{7}$$

Let us consider the dispersion properties of the wave propagating in the $z = 0$ plane. For that purpose we substitute the solution of Eqs. (5) into the boundary conditions (6) for the field above and below this plane. Above the conductive ionosphere in the semi-plane, where $\sigma_P = \sigma_H = 0$, we have

$$\frac{d^2E_y}{dz^2} - k^2E_y = 0,$$
$$\frac{d^2E_z}{dz^2} + 2ik\cot\varphi\frac{dE_z}{dz} - k^2\cot^2\varphi E_z = 0. \tag{8}$$

The solution for E_y, decaying at the infinity, and oscillatory solution for E_z take the form

$$E_y = a_2\exp(-kz), \quad E_z = b_2\exp(-ikz\cot\varphi) \tag{9}$$

In the region below the ionosphere $z<0$ corresponding to the insulated atmosphere we have $\Delta\mathbf{E} = 0$. The solutions for the electric field in this region reduce to

$$E_y = a_1\exp(kz), \quad E_z = b_1\exp(kz) \tag{10}$$

Substituting Eqs. (9) and (10) into the boundary conditions (6) one obtains the system of linear equations for the coefficients a_1, a_2, b_1 and b_2. Equating the determinant of this system to zero one finds the dispersion relation for the spectrum of eigen-oscillations given by

$$\left(2k^3l\cos^2\varphi - \frac{\omega^2}{a^2} - i\frac{\nu\omega}{a^2}k^2\cos^2\varphi\right)\times\left[kl + i\left(kl\cot\varphi - \frac{\nu\omega}{a^2\sin^2\varphi}\right)\right] - i\frac{\omega^2}{a^2}kl\left(kl\cot\varphi - \frac{\nu\omega}{a^2\sin^2\varphi}\right) = 0 \tag{11}$$

Eq. (11) represents the dispersion relation of gyrotropic waves propagating in $z = 0$ plane.

Let us now consider the influence of the conjugate ionosphere on the properties of the gyrotropic waves. For that purpose we use the model of the thin conjugate

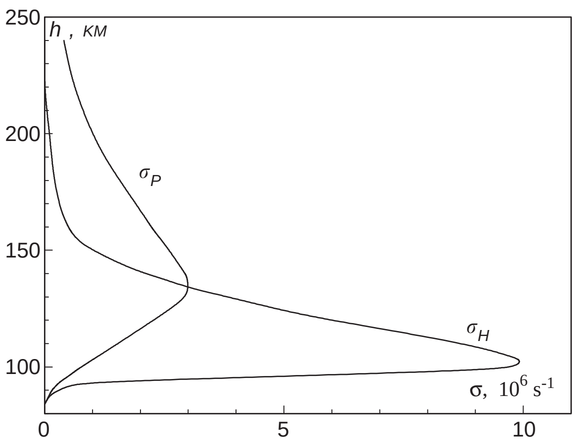

Fig. 1. Altitude dependence of the Hall and Pedersen conductivities.

ionosphere (e.g., Lyatskii and Maltsev, 1983). According to this model the conjugate ionosphere can be considered as a thin layer at the altitude $z = L$, as it is insured in the Fig. 2, where L corresponds to the distance between the conjugate ionospheres. We assume that at $z = 0$ the boundary condition (6) holds. At the conjugate plane $z = L$ the external magnetic field is directed under the angle $\psi = \pi - \varphi$ to the x-axis. Similar to Eqs. (6) one finds

$$\left\{\frac{\mathrm{d}E_y}{\mathrm{d}z}\right\}_{z=L} + \frac{\omega^2 + \mathrm{i}\nu\omega k^2\cos^2\psi}{k^2a^2l\cos^2\psi}E_y(L) = -\frac{2\omega\sin\psi}{ka\cos^2\psi}\frac{\mathrm{d}E_z(z=L+0)}{\mathrm{d}z}; \quad \{E_y\}_{z=L} = 0,$$
$$\left\{\frac{\mathrm{d}E_z}{\mathrm{d}z}\right\}_{z=L} = -\mathrm{i}\frac{\nu\omega}{a^2l\sin^2\psi}E_z(z=L-0),$$
$$\{E_z\}_{z=L} = -\frac{\omega}{2ka\sin\psi}E_y(L). \tag{12}$$

In the region $z<0$ the electric field components are defined by Eq. (10). In the region $0<z<L$, coinciding with the uniform magnetosphere, E_z yields to Eq. (11), the solution of which is

$$E_z = (b_2 + b_3 z)\exp(-\mathrm{i}kz\cot\varphi). \tag{13}$$

For $z>L$, i.e. in the conjugate atmosphere, one obtains

$$E_y = a_4\exp[-k(z-L)], \quad E_z = b_4\exp[-k(z-L)]. \tag{14}$$

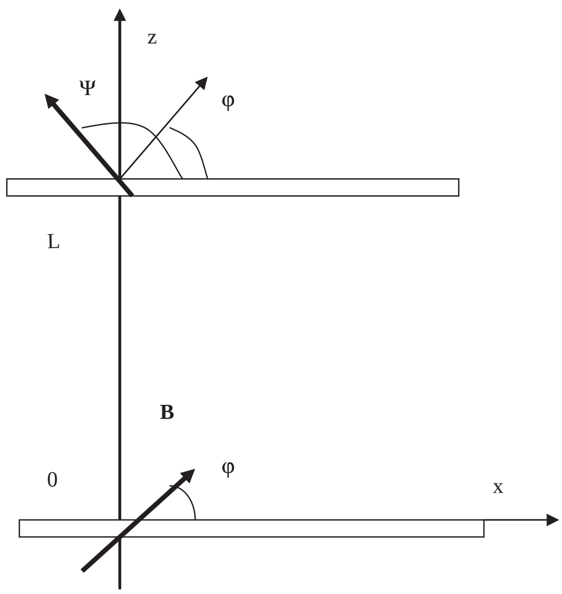

Fig. 2. Schematic drowing of the model. The planes $x = 0$ and $x = L$ coinside with the thin conjugate conductive layers. φ is the magnetic field inclination and ψ corresponds to the conjugate ionosphere.

The E_y component in the region $0<z<L$ satisfies the first equation in Eq. (11), the solution of which is

$$z>0, \quad E_y = a_2\exp(-kz),$$
$$z<L, \quad E_y = a_3\exp[k(z-L)]. \tag{15}$$

Substituting Eqs. (10) and (13)–(15) into the boundary conditions (6) and (12) one finds the system of linear equations for the coefficients $a_1 - a_4$ and $b_1 - b_4$. By setting the determinant of this system to zero one finds the dispersion relation

$$\left(2k^3l\cos^2\varphi - \frac{\omega^2}{a^2} - \mathrm{i}\frac{\nu\omega}{a^2}k^2\cos^2\varphi\right) \times \left[kl + \frac{2l}{L} + i\left(kl\cot\varphi - \frac{\nu\omega}{a^2\sin^2\varphi}\right)\right] - \frac{\omega^2}{a^2}kl\left[\frac{2l}{L} + i\left(kl\cot\varphi - \frac{\nu\omega}{a^2\sin^2\varphi}\right)\right] = 0. \tag{16}$$

Eq. (16) is the dispersion relation for gyrotropic waves accounting for the effect of the conjugate ionosphere. In the limiting case $kl \ll 1$ and $kL \gg 1$ this relation reduces to

$$\omega^2 + \mathrm{i}\omega k^2\nu\cos^2\varphi - 2k^3a^2l\cos^2\varphi = 0, \tag{17}$$

where the quantity ν (it is proportional to Pedersen conductivity) describes the damping of the gyrotropic waves.

Let us introduce the complex refractive index $\tilde{n} = n + i\kappa$ according to the formula $k = (n + \mathrm{i}\kappa)\omega/a$. Substituting this equality into Eq. (17) and assuming that the waves are weakly damping one finds the dependence of phase velocity $v_\mathrm{p} = a/n(\omega)$ and damping rate $\varepsilon = \lambda/2\pi\delta = \kappa/n$ as the function of frequency

$$v_\mathrm{p} = (2la^2)\omega^{1/3}\cos^{2/3}\varphi,$$
$$\varepsilon = \kappa/n = (\nu/6)(2/l^2a^4)^{1/3}\omega^{1/3}\cos^{2/3}\varphi. \tag{18}$$

Fig. 3 shows the dependence of $v_\mathrm{p} = v_\mathrm{p}(\omega)$ versus frequency for different magnetic inclination φ. For numerical calculations the following parameters have been selected: $l = 3\times10^6$ cm, $\sigma_\mathrm{H}(0) = 8\times10^6\,\mathrm{s}^{-1}$ and $\sigma_\mathrm{P}(0) = 2\times10^6\,\mathrm{s}^{-1}$. With these parameters in mind one finds: $a \approx c^2/4\pi l\sigma_\mathrm{H}(0) = 3\times10^6$ cm/s and $\nu = 2\times10^{12}\,\mathrm{cm}^2/\mathrm{s}$. Eq. (18) shows that the phase velocity of the gyrotropic waves and their specific damping increases with the growth of frequency as $\omega^{1/3}$. With the increase in the magnetic field inclination the wave phase velocity decreases. In the frequency range from 0.001 to 1 Hz the phase velocity varies from a few units to a few tenths of km /s.

3. Spatio-temporal characteristics of gyrotropic waves

Now we consider the spatio-temporal properties of geomagnetic pulsations generated by the gyrotropic

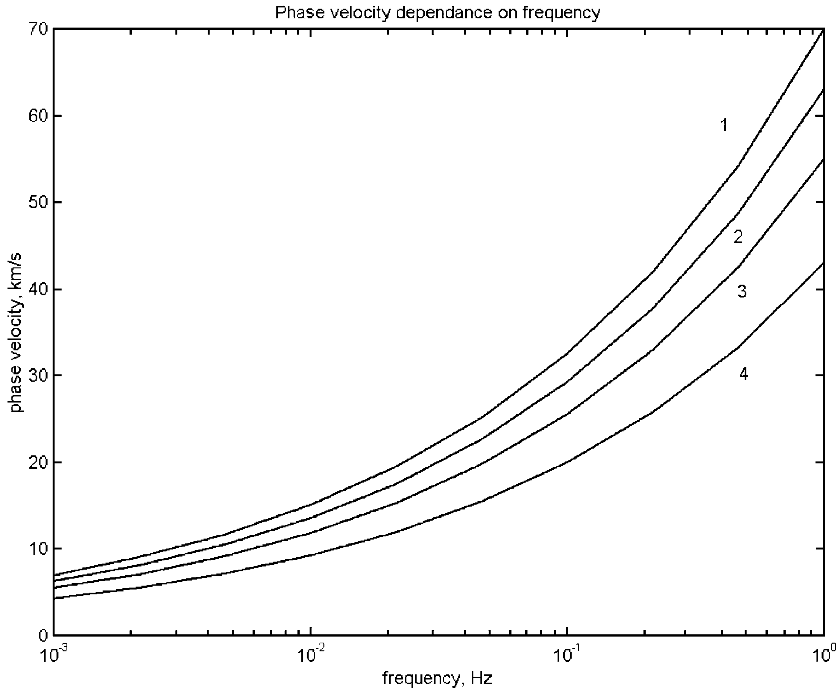

Fig. 3. Phase velocity of gyrotropic wave as a function of frequency for different values of φ. (1) $\varphi = 0$, (2) $\varphi = \pi/6$, (3) $\varphi = \pi/4$ and (4) $\varphi = \pi/3$. The Hall and Pedersen conductivities are: $\sigma_H(0) = 8 \times 10^6\,\mathrm{s}^{-1}$ and $\sigma_P(0) = 2 \times 10^6\,\mathrm{s}^{-1}$. The depth of the conductive layer $l = 3 \times 10^6$ cm, $a \approx c^2/4\pi l\sigma_H(0) = 3 \times 10^6$ cm/s and $v = 2 \times 10^{12}\,\mathrm{cm}^2/\mathrm{s}$.

wave in the ionospheric E-layer. The Fourier-components of the magnetic and electric fields are connected through Faradey's law

$$b_x(k,\omega) = \frac{c}{\mathrm{i}\omega}\left.\frac{\mathrm{d}E_y(k,\omega,z)}{\mathrm{d}z}\right|_{z=0}.$$

The magnetic field can be written as

$$b_x(k,t) = D(k)\exp\{-\mathrm{i}\omega_1(k)t\},$$

where the amplitude $D(k)$ should be determined from the boundary condition and $\omega_1(k)$ is the solution of dispersion relation (17)

$$\omega_1(k) = -(\mathrm{i}/2)vk^2\cos^2\varphi + \omega_0(k)\cos\varphi,$$
$$\omega_0(k) = \sqrt{2|k|^3a^2l}.$$

The spatio-temporal distribution of the magnetic field is

$$b_x(x,t) = \int_{-\infty}^{\infty}\frac{\mathrm{d}k}{2\pi}D(k)\exp\{\mathrm{i}kx - \mathrm{i}\omega_1(k)t\}. \quad (19)$$

This expression describes the wave propagating in the positive x-direction. Let at $x = x_0$ the temporal dependence be $b_x(x_1,t) = b_0(t)$, the Fourier-component of $b_0(t)$ is

$$b_0(\omega) = \int_{-\infty}^{\infty}\mathrm{d}t\,b_0(t)\exp(\mathrm{i}\omega t). \quad (20)$$

Setting in Eq. (19) $x = x_0$ and making a Fourier transform one obtains

$$b_0(\omega) = \int_{-\infty}^{\infty}\mathrm{d}k\,D(k)\exp(\mathrm{i}kx_0)\delta(\omega_1(k)-\omega).$$

From this expression one can obtain $D(k)$. Substituting it into Eq. (19) we have

$$b_x(x,t) = \int_{-\infty}^{\infty}\frac{\mathrm{d}k}{2\pi}\frac{\partial\omega_1(k)}{\partial k}b_0(\omega_1(k))\exp\{\mathrm{i}k(x-x_0) - \mathrm{i}\omega_1(k)t\}. \quad (21)$$

Expression (21) describes propagation of the magnetic field along the x-axis from the point x_0.

Let us consider the propagation of the wave packet along the x-axis. Let at $x = x_0$ the magnetic field be

$$b_0(t) = A(t)\sin(\Omega t),$$

where the amplitude $A(t)$ slowly varies in time. The Fourier transform of Eq. (20) gives

$$b_0(\omega) = \{A(\omega + \Omega) + A(\omega - \Omega)\}/2i.$$

The spatio-temporal distribution (21) in this case represents the sum over positive and negative values of k

$$b_x(x,t) = \frac{1}{4\pi \mathrm{i}}\left\{\int_{-\infty}^{0} \mathrm{d}k\,\frac{\partial\omega_1}{\partial k} A(\omega_1 + \Omega)\exp[\mathrm{i}k(x - x_0) - \mathrm{i}\omega_1 t] - \int_{0}^{\infty} \mathrm{d}k\,\frac{\partial\omega_1}{\partial k} A(\omega_1 - \Omega)\exp[\mathrm{i}k(x - x_0) - \mathrm{i}\omega_1 t]\right\}.$$

Let us consider the second integral in the complex plane $k = \xi + \mathrm{i}\varsigma$. Rapidly varying integrand A attains the maximum value at $k = k_0$ that is determined from the equation $\omega_1(k_0) = \Omega$. For small damping, $\varsigma_0 \ll \xi_0$, the latter condition gives

$$\Omega = \omega_0(\xi_0)\cos\varphi = \sqrt{2a^2 l \xi_0^3},$$
$$\varsigma_0 = v\xi_0^2 \cos^2\varphi / 2v_\mathrm{g},$$

where

$$v_\mathrm{g} = \partial\omega_1(k)/\partial k_{k\approx\xi_0} = (3/2)\cos\varphi\sqrt{2a^2 l\xi_0}.$$

The damping is small if $v\xi_0\cos^2\varphi \ll 2v_\mathrm{g}$. Taking this into account we expand ω_1 in the power series on $\xi - \xi_0$. In the leading order one obtains

$$\omega_1 \approx \Omega + v_g(\xi - \xi_0).$$

Similar expansion one can made in the first integral. Taking into account that in the vicinity of ξ_0 A is nonzero we have

$$b_x(x,t) = A\left(t - \frac{x - x_0}{v_\mathrm{g}}\right)\exp\left(-\frac{x - x_0}{\delta}\right) \times \sin\left[\Omega\left(t - \frac{x - x_0}{v_\mathrm{p}}\right)\right], \quad (22)$$

where $v_\mathrm{p} = \Omega/\xi_0$, $\delta = 2v_\mathrm{g}/v\xi_0^2\cos^2\varphi$. The phase and group velocities can be expressed in terms of Ω according to

$$v_\mathrm{p} = (2a^2 l\Omega)^{1/3}\cos^{2/3}\varphi, \quad v_\mathrm{g} = (3/2)v_\mathrm{p}. \quad (23)$$

Fig. 4 illustrates the propagation of the wave packet described by Eqs. (22) and (23). The function $A(t) at x = x_0$ is taken in the form

$$A(t) = b_0\exp[(t/T)^2].$$

The duration of the wave packet is $T = 15\,\mathrm{s}$ and the wave packet central frequency is $\Omega/2\pi = 0.1\,\mathrm{Hz}$. The curves 2 and 3 shows the perturbation of the magnetic field at the distances 1000 and 2000 km for the magnetic field inclinations $\pi/6$ and $\pi/3$, respectively.

Now we consider the propagation of the impulse of the gyrotropic waves. We assume that at $x = 0$ the magnetic field is given by $b_0(t) = b_0\delta(t)$, where $\delta(t)$ is the Dirac delta function. Substituting this condition into Eqs. (20) and (21) reduces to

$$b_x(x,t) = \frac{b_0 3a\sqrt{l}\cos\varphi}{\sqrt{2\pi}}\int_0^\infty \mathrm{d}k\sqrt{|k|}\exp\left(-\frac{vt}{2}k^2\cos^2\varphi\right) \times \cos\left[k\left(x - at\sqrt{2l|k|}\cos\varphi\right)\right]. \quad (24)$$

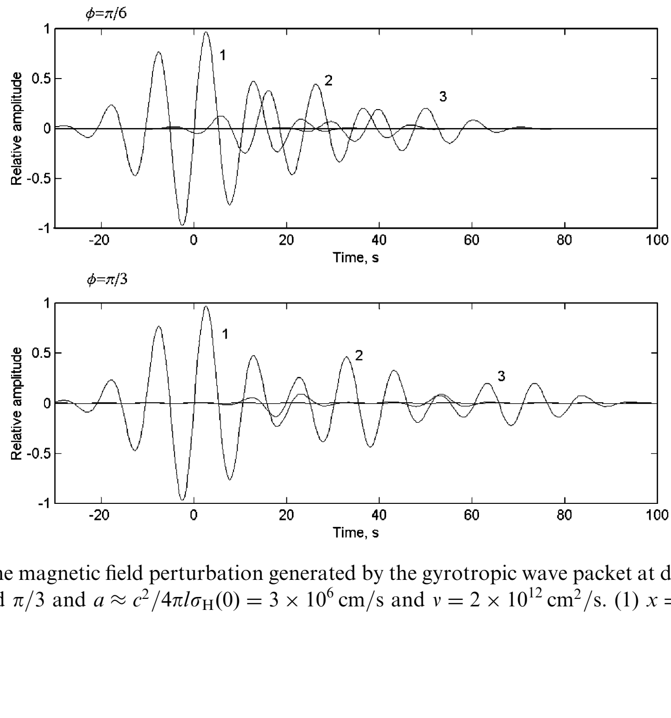

Fig. 4. Time dependence of the magnetic field perturbation generated by the gyrotropic wave packet at different distances for magnetic field inclinations $\varphi = \pi/6$ and $\pi/3$ and $a \approx c^2/4\pi l\sigma_\mathrm{H}(0) = 3\times 10^6\,\mathrm{cm/s}$ and $v = 2\times 10^{12}\,\mathrm{cm^2/s}$. (1) $x = 1000\,\mathrm{km}$, (2) $x = 2000\,\mathrm{km}$.

The condition of small damping allows us to use the stationary phase method for the estimation of the integral. Let us expand the function $f = k\left(x - at\sqrt{2l|k|}\cos\varphi\right)$ in power series on $k - k_0$ accounting for the small terms up to the second order

$$f(k) = f(k_0) + \tfrac{1}{2}f''(k_0)(k - k_0)^2. \quad (25)$$

The point of stationary phase k_0 and the expansion terms (25) are defined by

$$\frac{\mathrm{d}f(k)}{\mathrm{d}k_{k=k_0}} = 0, \quad k_0 = \frac{2x^2}{9t^2a^2l\cos^2\varphi},$$

$$f(k_0) = \frac{2x^3}{27t^2a^2l\cos^2\varphi}, \quad f''(k_0) = -\frac{9t^2a^2l\cos^2\varphi}{4x}. \quad (26)$$

Substituting Eqs. (25) and (26) into Eq. (24), one obtains

$$b_x(x,t) = b_0\frac{\sqrt{2}}{3a\sqrt{\pi l}\cos\varphi}\frac{x^{3/2}}{t^2} \times \exp\left(-\frac{2vx^4}{3^4t^3a^4l^2\cos^2\varphi}\right)\cos\left(\frac{2x^3}{3^3t^2a^2l\cos^2\varphi} - \frac{\pi}{4}\right). \quad (27)$$

Fig. 5 shows the time dependence of the magnetic field perturbations at the distances 1000 and 2000 km, respectively. The magnetic field inclinations are $\pi/6$ and $\pi/3$, respectively.

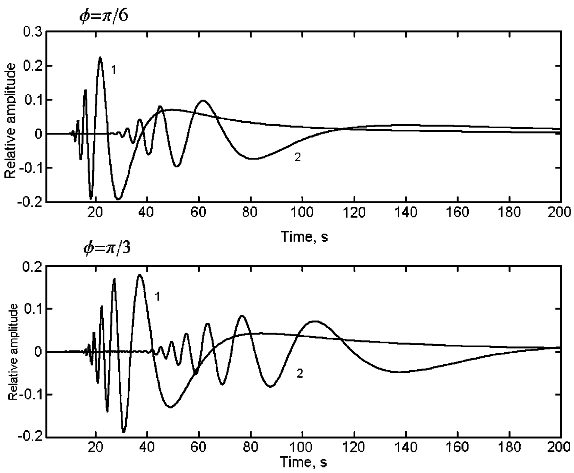

Fig. 5. Time dependence of the magnetic field perturbation generated by narrow unipolar impulse of the gyrotropic waves at different distances for magnetic field inclinations $\varphi = \pi/6$ and $\pi/3$. Other parameters are same as in Fig. 4.

4. The role of magnetic inclination in the formation geomagnetic pulsation spectra by gyrotropic waves

Let us consider the influence of the magnetic field inclination on the characteristics of the spectra of geomagnetic pulsations in the ULF frequency range. The latter can arise as the result of scattering of the background electromagnetic field on horizontal inhomogeneities of the ionospheric conductivity. Recently Sorokin et al. (2003) considered the formation of such spectra on the Earth's surface at low latitudes. It has been shown that this mechanism is due to generation of the gyrotropic waves in the low ionosphere by polarization currents that arise in the horizontal inhomogeneities of the conductivity under the action of the background electromagnetic field from the atmospheric sources. The calculations of the spectra were carried out with the use of the model of the wave propagation in the horizontal magnetic field. Below we assume that the waves propagate along the x-axis. The magnetic field is assumed to be in the $(x,\ z)$ plane. The conductivity tensor is taken in the form $\hat{\sigma}(x,z) = \hat{\sigma}_0(z) + \hat{\sigma}_1(x,z)$, where the subscripts 0 and 1 correspond to the unperturbed and perturbed values, respectively. We decompose the electric field as $\mathbf{E} = \mathbf{E_0} + \mathbf{E_1}$, where $\mathbf{E_0}$ stands for the unperturbed electric filed $(\hat{\sigma}_1 = 0)$ and $\mathbf{E_1}$ is the perturbation caused by the presence of the ionosphere inhomogeneities. Assuming the perturbations to be small, $\left|\hat{\sigma}_1\right| \ll \left|\hat{\sigma}_0\right|$ and neglecting the small terms of the second order from Eq. (5) one finds the electric field Fourier components as

$$E_{y1}(\omega,k) = -E_{y0}(\omega)\frac{2\omega^2}{\omega^2 - \omega_0^2(k,\varphi) + \mathrm{i}v\omega k^2\cos^2\varphi} \times \int_{-\infty}^{\infty}\mathrm{d}x\,\exp(\mathrm{i}kx)h(x);\ \omega_0^2(k,\varphi) = 2la^2|k|^3\cos^2\varphi,$$
$$h(x) = \sigma_{\mathrm{H1}}(x,z)/\sigma_{\mathrm{H0}}(z). \quad (28)$$

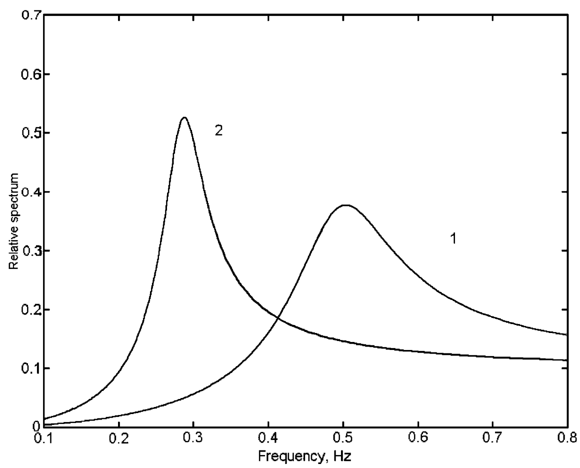

Fig. 6. Power spectra of geomagnetic pulsations on the Earth's surface generated by the gyrotropic waves in the presence of horizontal inhomogeneities of the ionospheric conductivity. (1) $\varphi = \pi/6$ and (2) $\varphi = \pi/3$, respectively. The parameters used: horizontal scale of the inhomogeneities $\lambda = 2\pi/k_0 = 100$ km, horizontal dimension of the perturbed region $L = 500$ km, $a \approx c^2/4\pi l\sigma_{\mathrm{H}}(0) = 3\times10^6$ cm/s and $v = 2\times10^{12}$ cm^2/s.

Let us take the perturbation of the Hall conductivity in the form $h(x) = A\exp(-|x|/L)\cos(k_0 x)$, where $L \gg \lambda = 2\pi/k_0$. Applying the inverse Fourier transform over k with the help of Faradey's law one obtains

$$\left|\frac{b_{1x}(x=0,\omega)}{b_{0x}(\omega)}\right| = A\left|\frac{\omega^2}{\omega^2 - \omega_0^2(k_0,\varphi) + \mathrm{i}\Phi^2(\omega,\varphi)\cos^2\varphi}\right|,$$
$$\Phi^2(\omega,\varphi) = \nu k_0^2\omega + 6la^2k_0^2/L. \tag{29}$$

Fig. 6 shows the power spectra for the angles $\pi/6$ and $\pi/3$, calculated with the help of Eq. (29). The parameters used are: $\lambda = 2\pi/k_0 = 100\,\mathrm{km}$ and $L = 500\,\mathrm{km}$. The frequencies corresponding to the maxima of the spectra lies in the frequency range 0.1–1 Hz. With the increase in the magnetic field inclination and horizontal scale of the inhomogeneity the frequencies of the maxima become smaller.

5. Discussion and conclusions

In the present paper it has been shown that the Earth's ionosphere can support a specific class of electromagnetic wave modes termed the gyrotropic waves. They represent the weakly damping oscillations of the electric current and the electric field and propagate in the thin ionospheric layer. The magnetic field induced by this current can be observed on the Earth's surface as geomagnetic pulsations whereas the electric field forms the ionospheric perturbations in the F-layer. The phase velocity of these waves lies in the frequency range 0.001–1 Hz and amounts for the values stretching from a few units to a few tenths of km/s. It decreases as $\cos^{2/3}\varphi$ with the increase in the magnetic field inclination. In the mid- and high-latitudes the phase velocity is smaller than in the low latitudes. The damping of these waves possess a similar latitudinal dependence. The frequency dependence of the phase velocity varies as $\omega^{1/3}$. The high-frequency harmonics of the impulse take the lead over the low-frequency harmonics. The wave phase velocity decreases with the growth of Hall conductivity. Thus, it is greater in the nighttime ionosphere. The wave damping is controlled by Pedersen conductivity.

The analysis of the influence of the magnetic field inclination and Pedersen conductivity on propagation of the wave packet has shown that at the distance of 1000 km its amplitude is two times smaller whereas at the distance 2000 km it is three times smaller. The wave damping constitutes the value $\kappa/n = \lambda/2\pi\delta = 10^{-1}–10^{-2}$. The phase velocity decreases with the growth of the wave period and magnetic field inclination. The wave packet propagates with the group velocity exponentially decreasing with the amplitude without significant distortion of the form. The characteristic spatial scale of the amplitude variation constitutes the value of 2000–3000 km. The phase velocity exceeds the group velocity. The obtained propagation characteristics of the quasi-harmonic wave packet agree with observations of the phase shift of geomagnetic pulsations observed at the array of spatially displaced magnetometers. In the course of propagation of the short unipolar impulse it has been observed the enhancement of its duration and appearance of oscillations. Their frequency decreases with time. The signal duration increases with the magnetic field inclination. The formation of the ionospheric horizontal periodic inhomogeneities of the conductivity leads to the appearance of geomagnetic pulsations on the Earth's surface. These oscillations are associated with the gyrotropic waves. The waves are generated by the electric current that arises in the inhomogeneities of the ionospheric conductivity under the action of the external electric field from the atmospheric sources. The spectrum of oscillations possesses the maximum in the ULF frequency range. The inhomogeneities can arise, for example, during upward propagation of gravitational waves from seismic sources. If the inhomogeneities are elongated in the latitudinal direction and their horizontal scale is of the order of 100 km the central spectrum frequency lies in the range 0.1–1 Hz. The central frequency decreases with the increase in magnetic field inclination and spatial scale of the inhomogeneity. The considered mechanism can be used for the interpretation of the observed phase shift at the Earth's surface and dispersion of the ULF waves, connected with the growth of seismic activity. The horizontal phase velocity of these oscillations constitutes the values 20–30 km/s. The features of the gyrotropic waves can be used for the interpretation of the electromagnetic effects that arise in the course of natural and man-made action on the ionosphere.

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Appendix

Fig. 1 shows the variation of σ_P and σ_H conductivities versus altitude. One sees that both conductivities attain the maximum values at different altitudes. The maximum of Hall conductivity lies higher than that for Pedersen conductivity. The maximum value of Hall conductivity exceeds that for Pedersen conductivity by one–two orders in value. At the higher altitudes the Pedersen conductivity substantially exceeds Hall conductivity. Such an altitude profile allows us to represent the ionosphere in the form of two distinct thin layers. Let us assume that in the low layer $\sigma_P = 0$ whereas in the upper layer $\sigma_H = 0$. In the low frequency limit, $\omega \ll \sigma_{P,H}$, we assume all perturbed values to vary as $\exp(ikx - i\omega t)$. The system of Eqs. (5) can be written independently in each of these layers. In the low layer, in which Hall conductivity is nonzero, one has

$$\frac{d^2E_y}{dz^2} - k^2E_y = i\frac{4\pi\omega\sigma_H}{c^2 \cos\varphi}E_z,$$
$$\sin^2\varphi\frac{d^2E_z}{dz^2} + 2ik\sin\varphi\cos\varphi\frac{dE_z}{dz} - k^2\cos^2\varphi E_z = -i\frac{4\pi\omega\sigma_H\cos\varphi}{c^2}E_y. \tag{A.1}$$

In the upper layer, where Pedersen conductivity is nonzero, one obtains

$$\frac{d^2E_y}{dz^2} - k^2E_y = -i\frac{4\pi\omega\sigma_P}{c^2}E_y,$$
$$\sin^2\varphi\frac{d^2E_z}{dz^2} + 2ik\sin\varphi\cos\varphi\frac{dE_z}{dz} - k^2\cos^2\varphi E_z = -i\frac{4\pi\omega\sigma_P}{c^2}E_z. \tag{A.2}$$

Let us assume that the horizontal scale of the wave field variation is much greater than the depth of the conductive layer l, i.e. $kl \ll 1$. In this case Eqs. (A.1) and (A.2) can be replaced by the boundary conditions for the electric field components and their derivatives. Let us find the boundary conditions in the lower layer coinciding with the $z = 0$ plane. Substituting E_z from the second Eq. of the system (A.1) into the second equation and integrating over the layer we obtain

$$\left\{\frac{dE_y}{dz}\right\}_{z=0} - k^2\int_{-l/2}^{l/2} dzE_y + \left(\frac{4\pi\omega}{c^2k\cos\varphi}\right)^2\int_{-l/2}^{l/2} dz\sigma_H^2E_y = -\frac{8\pi\omega\sin\varphi}{c^2k\cos^2\varphi}\int_{-l/2}^{l/2} dz\sigma_H\frac{dE_z}{dz} + i\frac{4\pi\omega\sin^2\varphi}{c^2k^2\cos^3\varphi}\int_{-l/2}^{l/2} dz\sigma_H\frac{d^2E_z}{dz^2},$$

$$\sin^2\varphi\left\{\frac{dE_z}{dz}\right\}_{z=0} + 2ik\sin\varphi\cos\varphi\{E_z\}_{z=0} - k^2\cos^2\varphi\int_{-l/2}^{l/2} dzE_z = -i\frac{4\pi\omega\cos\varphi}{c^2}\int_{-l/2}^{l/2} dz\sigma_HE_y,$$

where the parentheses denote the difference between the values above and below the layer. The integrand denotes the multiplication of rapidly varying function σ_H that attains the maximum value at $z = 0$ and slowly varying along the layer function. From this system one sees that in the plane $z = 0$ the following conditions should be satisfied

$$\{E_y\}_{z=0} = 0, \quad \left\{\frac{dE_y}{dz}\right\}_{z=0} + \frac{\omega^2}{k^2a^2l\cos^2\varphi}E_y(0) = -\frac{2\omega\sin\varphi}{ka\cos^2\varphi}\frac{dE_z(0)}{dz}\left\{\frac{dE_z}{dz}\right\}_{z=0} = 0,$$
$$\{E_z\}_{z=0} = -\frac{\omega}{2ka\sin\varphi}E_y(0). \tag{A.3}$$

In Eq. (A.3) the following abbreviations are introduced

$$a = c^2\Big/4\pi\sqrt{l\int_{-\infty}^{\infty} dz\sigma_H^2(z)}, \quad l\sigma_H^2(0) = \int_{-\infty}^{\infty} dz\sigma_H^2(z).$$

The boundary condition (A.3) for E_z one can obtain from the solution of the second Eq. (A.1) in the layer. Letting $E_z(z) = A(z)\exp(-ikz\cot\varphi)$, one finds

$$\frac{d^2A}{dz^2} = -i\frac{4\pi\omega\cos\varphi}{c^2\sin^2\varphi}\sigma_HE_y\exp(ikz\cot\varphi).$$

Inside the layer one has

$$E_z(z) = \left[E_z\left(-\frac{l}{2}\right)\exp\left(-i\frac{kl}{2}\cot\varphi\right) + \left(z + \frac{l}{2}\right)C\right] \times \exp(-ikz\cot\varphi) - i\frac{4\pi\omega\cos\varphi}{c^2\sin^2\varphi}\int_{-l/2}^{z} dz' \int_{-l/2}^{z'} dz''\sigma_H(z'')E_y(z'')\exp[ik(z''-z)\cot\varphi].$$

The constant C is defined from the condition of continuity of the derivative

$$\left\{\frac{dE_z}{dz}\right\}_{z=0} = \lim_{l\to 0}\left[\frac{dE_z}{dz|_{z=1/2}} - \frac{dE_z}{dz|_{z=-1/2}}\right] = 0.$$

Defining the constant, for $kl\cot\varphi 1$, one obtains

$$E_z\left(\frac{l}{2}\right) - E_z\left(-\frac{l}{2}\right) = -\frac{2\pi\omega}{c^2k\sin\varphi}\int_{-l/2}^{l/2} dz\sigma_H(z)E_y(z) \times \left[ik\left(\frac{l}{2} - z\right)\cot\varphi + 1\right]\exp\left[-ik\left(\frac{l}{2} - z\right)\cot\varphi\right].$$

Substituting $\sigma_H(z) = \sigma_H(0)l\delta(z)$, where $\delta(z)$ is the Dirac delta function, for $kl\cot\varphi \ll 1$, one obtains

$$\{E_z\}_{z=0} = -\frac{2\pi\omega}{c^2k\sin\varphi}\sigma_H(0)lE_y(0)\left(ik\frac{l}{2}\cot\varphi + 1\right) \times \exp\left(-ik\frac{l}{2}\cot\varphi\right) \approx -\frac{\omega}{2ka\sin\varphi}E_y(0)$$

This expression coincides with Eq. (A.3). Thus, the boundary condition for the vertical component of the electric field is valid if the condition $\sin\varphi \gg kl$ is satisfied. Let the layer in which Pedersen conductivity is nonzero lie in the $z = z_0$ plane. Integrating Eq. (A.2) over the layer, one obtains

$$\left\{\frac{\mathrm{d}E_y}{\mathrm{d}z}\right\}_{z=z_0} = -\mathrm{i}\frac{\omega}{a_\mathrm{P}}E_y(z_0),$$
$$\left\{\frac{\mathrm{d}E_z}{\mathrm{d}z}\right\}_{z=z_0} = -\mathrm{i}\frac{\omega}{a_\mathrm{P}\sin^2\varphi}E_z(z_0),$$
$$\{E_y\}_{z=z_0} = 0, \quad \{E_z\}_{z=z_0} = 0,$$
$$a_\mathrm{P} = c^2/4\pi\int_{-\infty}^{\infty}\mathrm{d}z\sigma_\mathrm{P}(z). \tag{A.4}$$

Setting z_0 to zero and summing up the Eqs. (A.3) and (A.4), one obtains the boundary conditions for the components of the electric field on the conductive thin ionosphere

$$\left\{\frac{\mathrm{d}E_y}{\mathrm{d}z}\right\}_{z=0} + \frac{\omega^2 + \mathrm{i}v\omega k^2\cos^2\varphi}{k^2a^2l\cos^2\varphi}E_y(0)$$
$$= -\frac{2\omega\sin\varphi}{ka\cos^2\varphi}\frac{\mathrm{d}E_z(z=-0)}{\mathrm{d}z}, \quad \{E_y\}_{z=0} = 0,$$
$$\left\{\frac{\mathrm{d}E_z}{\mathrm{d}z}\right\}_{z=0} = -\mathrm{i}\frac{v\omega}{a^2l\sin^2\varphi}E_z(z=+0),$$
$$\{E_z\}_{z=0} = -\frac{\omega}{2ka\sin\varphi}E_y(0), \tag{A.5}$$

where $v = a^2l/a_\mathrm{P}$.

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