JOURNAL OF GEOPHYSICAL RESEARCH, VOL. 113, A06306, doi:10.1029/2008JA013094, 2008

On the generation of narrow-banded ULF/ELF pulsations in the lower ionospheric conducting layer

V. M. Sorokin[1] and M. Hayakawa[2]

Received 13 February 2008; accepted 31 March 2008; published 20 June 2008.

[1] The coupling between the atmosphere and lower ionosphere (due to either the DC electric field or atmospheric gravity waves) may lead to the formation of horizontal irregularities of ionospheric conductivity over an earthquake preparation region. Such interaction of these irregularities with the background electromagnetic noise leads to occurrence of periodic polarization electric currents. These currents are considered to be a coherent source of gyrotropic waves propagating in the ionospheric conducting layer with finite thickness. The theoretical estimation has been performed on the generation of those ionospheric micropulsations in the frequency range of ULF to ELF in the equatorial latitude with horizontal Earth's magnetic field. Then it is found that those waves appear as a narrow-banded line spectrum in the frequency range from 1 to 30 Hz, and these waves are expected to be observed on Earth's surface.

Citation: Sorokin, V. M., and M. Hayakawa (2008), On the generation of narrow-banded ULF/ELF pulsations in the lower ionospheric conducting layer, *J. Geophys. Res.*, *113*, A06306, doi:10.1029/2008JA013094.

1. Introduction

[2] The ground-based measurements yielded the detection of discrete narrow-band spectra of the extremely low-frequency electromagnetic oscillations during either seismic enhancement, volcanic eruptions or spacecraft flights [*Rauscher and Van Bise*, 1999]. It was found that the spectrum maxima are located approximately at separate frequencies of 2, 6, 11, and 17 Hz, and these processes seem to be associated with the formation of horizontal irregularities of the ionospheric conductivities. *Afraimovich et al.* [2002] observed the disturbances of total electron contents by means of GPS technique during rocket starts, whose horizontal spatial scale is of the order of 50–100 km. Then the satellite data showed irregularities of electron number density with the scale over ten km in the ionosphere before an earthquake [*Chmyrev et al.*, 1997]. The occurrence of ionospheric irregularities over seismic regions was also confirmed in the F layer by *Afonin et al.* [1999] on the basis of the satellite density measurement and also in the lower ionosphere by *Rozhnoi et al.* [2004] and *Maekawa et al.* [2006] on the basis of subionospheric VLF/LF propagation data. These irregularities over seismic and volcanic regions might be caused by either upward acoustic-gravity wave propagation [*Molchanov et al.*, 2001; *Miyaki et al.*, 2002; *Mareev et al.*, 2002; *Rozhnoi et al.*, 2007; *Horie et al.*, 2007] or DC electric field enhancement in the ionosphere [*Sorokin et al.*, 1998].

[3] *Sorokin et al.* [2003] have found an ionospheric generation mechanism of geomagnetic pulsations as observed on Earth's surface by *Rauscher and Van Bise* [1999]. This mechanism is based on the excitation of gyrotropic waves [*Sorokin and Fedorovich*, 1982; *Sorokin and Pokhotelov*, 2005] by the coherent polarization electric currents located in the irregularities of ionospheric conductivity. These currents are generated by the background electromagnetic noise and various sources might lead to such an electromagnetic noise in ULF/ELF range, with the most powerful being thunderstorms. The oscillating noise electric field forms the polarization currents on the irregularities of ionospheric conductivity. Gyrotropic waves are propagated within a thin layer of the lower ionosphere along Earth's surface with small attenuation and with phase velocities of the order of tens to hundreds km/s.

[4] The present paper is devoted to the investigation on the generation and propagation of those gyrotropic waves in the conducting layer with a finite thickness of the lower ionosphere and the calculation of spectrum of magnetic field oscillations on Earth's surface related to these waves. Furthermore, we deal reasonably with the equatorial case in which Earth's magnetic field is horizontal, because powerful seismic activity takes place at lower latitudes.

2. Wave Equations and Boundary Conditions of an Electromagnetic Field in the Ionospheric Conducting Layer With Finite Thickness

[5] We consider the generation of gyrotropic waves owing to the occurrence of irregularities in conductivity in the presence of a background electromagnetic field in the lower ionosphere. A rather simplified situation is considered in Figure 1. The Earth's magnetic field is assumed to be horizontal and homogeneous, which corresponds to very

[1]Ionosphere and Radio Wave Propagation, Institute of Terrestrial Magnetism, Troitsk, Moscow Region, Russia.
[2]Research Station on Seismo Electromagnetics, Department of Electronic Engineering, University of Electro-Communications, Tokyo, Japan.

0148-0227/08/2008JA013094$09.00

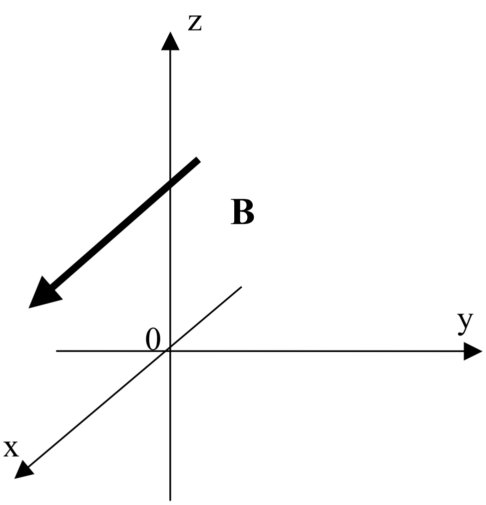

Figure 1. The configuration of the problem. Earth's magnetic field is assumed to be horizontal and homogeneous and is directed in the x axis. The Cartesian coordinate system is used (z, local vertical), and the wave propagates in the x direction.

low latitudes or equatorial case where we often encounter powerful earthquakes [e.g., *Hayakawa et al.*, 2006; *Horie et al.*, 2007]. The magnetic meridian plane is (x, z) plane and the magnetic field **B** is directed in the x direction. The z axis of our Cartesian coordinate system, is coincident with the local vertical direction. The lower ionosphere is assumed to be plane and horizontally stratified with the Hall conductivity, σ_H and Pedersen conductivity σ_P. The conductivity is given in the form of $\sigma_{P,H} = \sigma_{P0,H0} + \sigma_{P1,H1}$, where the subscripts 0 and 1 correspond to the unperturbed and perturbed values, respectively. The upper panel of Figure 2 illustrates the typical height profiles of the unperturbed σ_{P0} and σ_{H0} conductivities.

[6] We decompose the electric field and the perturbation of magnetic field as $\mathbf{E} = \mathbf{E}_0 + \mathbf{E}_1$ and $\mathbf{b} = \mathbf{b}_0 + \mathbf{b}_1$, where $\mathbf{E}_0$ and $\mathbf{b}_0$ stand for the electromagnetic field of background noise when $\widehat{\sigma}_1 = 0$, and $\mathbf{E}_1$ and $\mathbf{b}_1$ are their perturbations caused by the presence of the ionospheric inhomogeneities. Assuming the perturbations to be small $|\widehat{\sigma}_1| \ll |\widehat{\sigma}_0|$ and neglecting small terms of the second order in the ULF/ELF frequency region $\omega \ll 4\pi\sigma_{P,H} \approx 10^7$ s^{-1}, one finds the equations for the electric field perturbations as follows [*Sorokin et al.*, 2003].

$$\begin{aligned}(\nabla \times \nabla \times \mathbf{E}_1) \times \mathbf{B} + \frac{4\pi}{c^2}\frac{\partial}{\partial t}(\sigma_{P0}\mathbf{E}_1 \times \mathbf{B} - \sigma_{H0}B\mathbf{E}_1) \\ = \frac{4\pi}{c^2}\frac{\partial}{\partial t}(\sigma_{P1}\mathbf{E}_0 \times \mathbf{B} - \sigma_{H1}B\mathbf{E}_0)\nabla \times \mathbf{E}_1 \\ = -\frac{1}{c}\frac{\partial \mathbf{b}_1}{\partial t},\end{aligned} \quad (1)$$

[7] The ionospheric irregularities are assumed to be stretched along the y axis, and the spatial scale of conductivity variations in these irregularities is much larger than the temporal scale of the electromagnetic oscillations. Let E_{0y} be the horizontal component of electromagnetic background noise. The wave is assumed to propagate in the x direction, and the electromagnetic field of this wave has the transverse components, E_{1y} and E_{1z}, which depends on the coordinates, x and z. Then, the field component of the wave along the x axis is equal to zero, $E_{1x} = 0$. Using the conventional Fourier transformation in terms of the coordinate x and time t:

$$E_{1,x,y,z}(k, z, \omega) = \int_{-\infty}^{\infty} dx \int_{-\infty}^{\infty} dt E_{1,x,y,z}(x, z, t) \exp(-ikx + i\omega t).$$

We obtain from equation (1) the wave equations for the components of electric field disturbances:

$$\begin{aligned}k^2 E_{1z} + i\frac{4\pi\omega}{c^2}(\sigma_{H0}E_{1y} - \sigma_{P0}E_{1z}) \\ = -i\frac{4\pi\omega}{c^2}\sigma_{H0}f_H\left(\frac{d^2}{dz^2} - k^2\right)E_{1y} + i\frac{4\pi\omega}{c^2}(\sigma_{P0}E_{1y} + \sigma_{H0}E_{1z}) \\ = -i\frac{4\pi\omega}{c^2}\sigma_{P0}f_P E_{1x} = 0\end{aligned} \quad (2)$$

In equation (2) the following designations are introduced.

$$\begin{aligned}f_P(k, \omega) &= \int_{-\infty}^{\infty} P(x)E_{0y}(x, \omega)\exp(ikx)dx \\ f_H(k, \omega) &= \int_{-\infty}^{\infty} H(x)E_{0y}(x, \omega)\exp(ikx)dx \\ P(x) &= \sigma_{P1}(x, z)/\sigma_{P0}(z), H(x) = \sigma_{H1}(x, z)/\sigma_{H0}(z)\end{aligned} \quad (3)$$

For the solution of equation (2) we use the boundary conditions connecting the tangential components of the electrical field and its vertical derivative above and below the ionospheric conducting layer as shown in Figure 2b (see Appendix A).

$$\begin{aligned}E_{1y}\left(\frac{l}{2}\right) - \xi_1 E_{1y}\left(-\frac{l}{2}\right) - \xi_2\frac{d}{dz}E_{1y}\left(-\frac{l}{2}\right) \\ = \varsigma_1 f_H \frac{d}{dz}E_{1y}\left(\frac{l}{2}\right) - \xi_3\frac{d}{dz}E_{1y}\left(-\frac{l}{2}\right) - \xi_4 E_{1y}\left(-\frac{l}{2}\right) \\ = \varsigma_2 f_H - \varsigma_3 f_P\end{aligned} \quad (4)$$

The terms on the right-hand side of boundary conditions of equation (4) depend on conductivity disturbances. Furthermore we have to find the solutions of equation (2) in the region above and below the conducting layer and then substitute them in the boundary conditions.

3. Calculation of the Amplitude Frequency Dependence of Magnetic Field Disturbances

[8] The horizontal spatial scale of the background electric field exceeds considerably the spatial scale of conductivity irregularities in ULF/ELF frequency range. It means that the field varies slowly on the horizontal scale of the irregularities $E_{0y}(x, \omega) \approx E_{0y}(\omega)$. Hence, we have from equation (3),

$$f_P(k, \omega) = P(k)E_{0y}(\omega); \quad f_H(k, \omega) = H(k)E_{0y}(\omega),$$

where $P(k)$ and $H(k)$ are the Fourier components of relative irregularities of the ionospheric conductivities. The velocity of hydromagnetic waves in the magnetosphere considerably

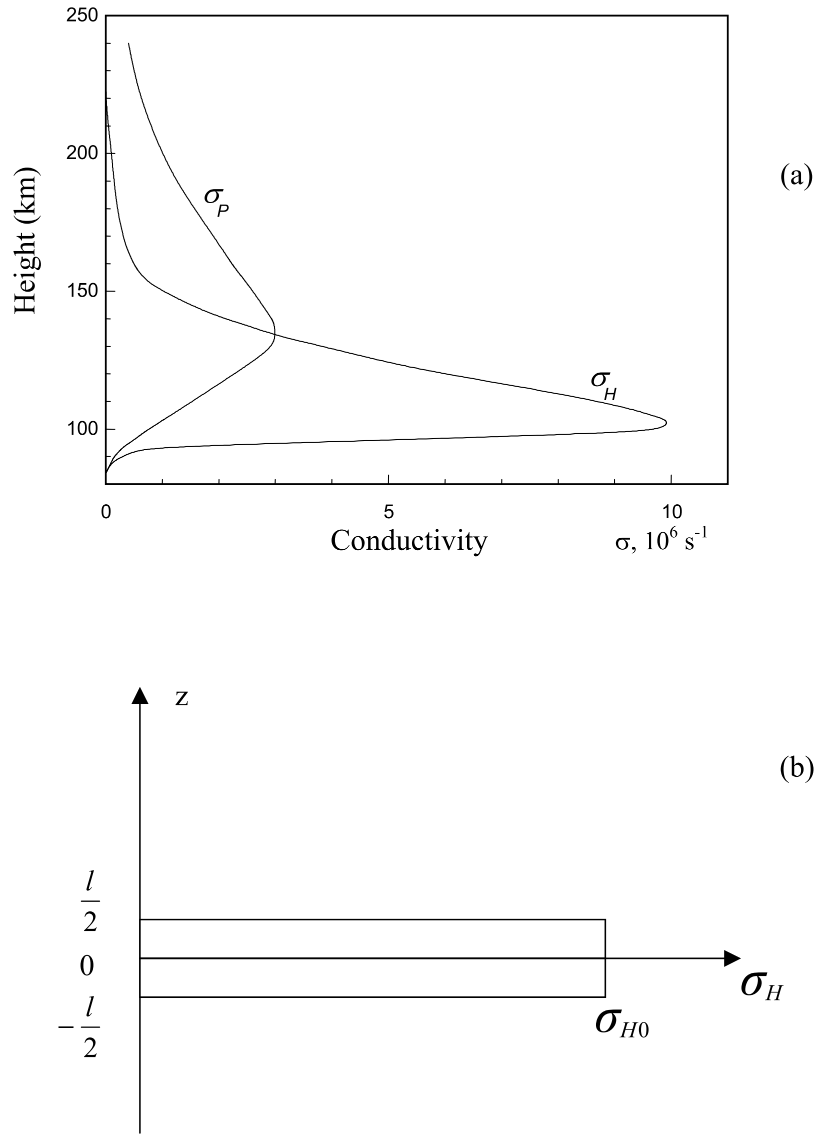

Figure 2. H(a) eight profiles of ionospheric Hall conductivity (σ_H) and Pedesen conductivity (σ_P) and (b) the approximated slab structure with conductivity σ_{H0} and with thickness of l.

exceeds one of gyrotropic waves in the E layer of ionosphere. Therefore, the electric field is determined from the Laplace equation $\Delta E_{1y} = 0$ both above and below the ionosphere. The solution of this equation has the form,

$$E_{1y} = A_1 \exp\left[-|k|\left(z - \frac{l}{2}\right)\right], z > \frac{l}{2};$$
$$E_{1y} = A_2 \exp\left[|k|\left(z + \frac{l}{2}\right)\right], z < -\frac{l}{2}. \qquad (5)$$

Substituting equation (5) into the boundary conditions equation (4), one finds the horizontal component of electric field disturbances on the ionospheric bottom border:

$$E_{1y} = \left(k, z = -\frac{l}{2}, \omega\right) = -E_{0y}(\omega)\frac{(|k|\varsigma_1 - \varsigma_2)H - \varsigma_3 P}{|k|\xi_1 + k^2\xi_2 + |k|\xi_3 + \xi_4} \qquad (6)$$

Functions ξ_i and ς_i are expressed in Appendix A. According to previous works both acoustic-gravity waves [*Molchanov et al.*, 2001; *Miyaki et al.*, 2002; *Horie et al.*, 2007; *Molchanov and Hayakawa*, 2008], and atmospheric electric currents [*Sorokin et al.*, 1998; *Sorokin*, 2007] lead to the formation of horizontal irregularities in the ionosphere conductivity above a seismic region. Their propagation speed is of the order of acoustic wave velocity u which is much smaller than that of gyrotropic waves. We choose the dependence of ionosphere conductivity irregularities on the coordinate x in the following form,

$$H(x) = P(x) = A_0 \exp\left(-x^2/4x_0^2\right)\cos(k_0 x),$$

where $k_0 = 2\pi/\lambda_0$, $\lambda_0 = uT$, the horizontal spatial scale of conductivity irregularities, T is the temporal scale of conductivity irregularities, $x_0 \gg \lambda_0$ is the horizontal spatial scale of the seismic region, and A_0 is the maximal value of

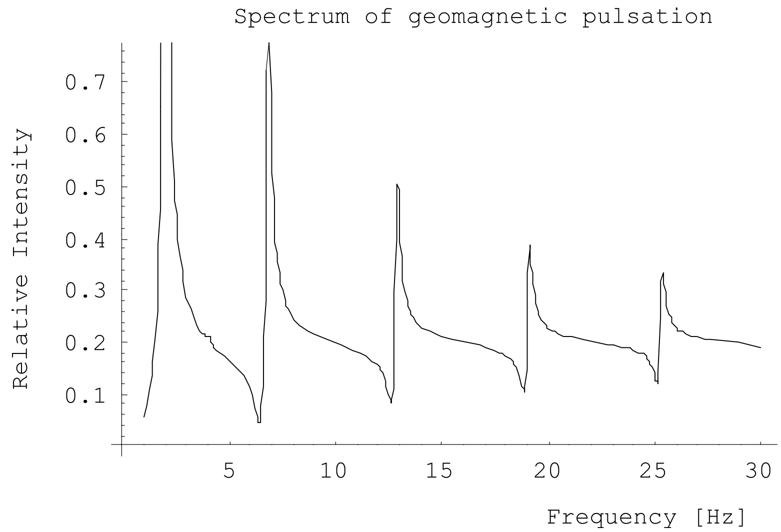

Figure 3. An example of computational results on the frequency spectrum in the ULF/ELF band of ionospheric pulsations to be observed on Earth's surface.

relative conductivity disturbances. The Fourier image of these disturbances has the following form,

$$H(k) = P(k) = \sqrt{\pi}x_0A_0\Big\{\exp\Big[-x_0^2(k_0-k)^2\Big] + \exp\Big[-x_0^2(k_0+k)^2\Big]\Big\}.$$

Substituting this equation into equation (6) and applying the inverse Fourier transform over k, we obtain,

$$\frac{E_{1y}(x, z-l/2, \omega)}{E_{0y}(\omega)} = -\frac{x_0A_0}{2\sqrt{\pi}}\int_{-\infty}^{\infty} dkF(k,\omega) \cdot \{\exp f_1(k,x) + \exp f_2(k,x)\} f_{1,2}(k,x) = ikx - (k_0 \pm k)^2 x_0^2. \quad (7)$$

Some functions in equation (7) are given below,

$$F(k,\omega) = \frac{g_1 q \sinh(ql) - g_2[1-\cosh(ql)] + g_3q^2}{G_1 q\sinh(ql) + G_2 q^2\cosh(ql)},$$
$$g_1 = \kappa_H^2;\ \ g_2 = \kappa_H^2(|k| - i\kappa_P);\ \ g_3 = i\kappa_P;$$
$$G_1 = \kappa_H^2 - 2k^2 + i\kappa_P|k|;\ \ G_2 = -2|k| + i\kappa_P$$
$$q^2 = k^2 - \kappa_H^2;\quad \kappa_H = 4\pi\omega\sigma_0/c^2k;\quad \kappa_P = 4\pi\omega\Sigma_P/c^2 \quad (8)$$

We derived formula (8) using the functions ξ_i and ς_i, which were obtained in Appendix A. The limit of $l \to 0$ and σ_0 under the condition of $\sigma_0^2 l = \int_{-\infty}^{\infty} \sigma_{H0}^2(z)dz = const$ in equation (8) lead to the model of an infinitely thin conducting layer of the ionosphere, which was considered by *Sorokin et al.* [2003]. On the basis of the Maxwell's equation $\nabla \times \mathbf{E} = (i\omega/c)\mathbf{b}$, we obtain an equality between relative disturbances of electric field on the bottom border of the ionosphere $z = -l/2$ and relative disturbances of magnetic field β on Earth's surface $z = -z_1$,

$$\beta(x,\omega) = \left|\frac{b_{1x}(x, z=-z_1, \omega)}{b_{0x}(\omega)}\right| = \left|\frac{E_{1y}(x, z=-l/2, \omega)}{E_{0y}(\omega)}\right|.$$

We have estimated the integral (7) by Laplace method replacing functions $f_{1,2}(k, x)$ by their factorization about the extreme points $k_{1,2}$ and leaving terms of the second-order smallness,

$$f_{1,2}(k,x) \approx ik_{1,2} - x_0^2(k-k_{1,2})^2;\quad k_{1,2} = \mp k_0 + i(x/2x_0^2). \quad (9)$$

Substituting equation (9) in equation (7), one finds,

$$\beta(x,\omega) = \frac{A_0}{2}\exp\left(-\frac{x^2}{4x_0^2}\right)\left|F\left(k_0 + i\frac{x}{2x_0^2},\omega\right)\exp(ik_0x) + F\left(k_0 - i\frac{x}{2x_0^2},\omega\right)\exp(-ik_0x)\right|. \quad (10)$$

Below we consider the relative spectrum of magnetic pulsations in the epicenter of a seismic region $x = 0$ on Earth's surface. We find from equation (10),

$$\beta(x=0,\omega) = \left|\frac{b_{1x}(\omega)}{b_{0x}(\omega)}\right| = A_0\left|\frac{g_1q\sinh(ql) - g_2[1-\cosh(ql)] + g_3q^2}{G_1q\sinh(ql) + G_2q^2\cosh(ql)}\right|, \quad (11)$$

where

$$g_1 = \kappa_H^2;\ \ g_2 = \kappa_H^2(|k_0| - i\kappa_P);\ \ g_3 = i\kappa_P;$$
$$G_1 = \kappa_H^2 - 2k_0^2 + i\kappa_P|k_0|;\ \ G_2 = -2|k_0| + i\kappa_P;$$
$$q^2 = k_0^2 - \kappa_H^2;\quad \kappa_H = 4\pi\omega\sigma_0/c^2k_0;\quad \kappa_P = 4\pi\omega\Sigma_P/c^2$$

[9] The calculation result of the spectrum obtained by using equation (11) is presented in Figure 3. In this computation we have used the following reasonable parameters,

$$A_0 = 0.20,\quad \sigma_{H0} = 2\times10^6\ \mathrm{s}^{-1},\quad l = 3\times10^6\ \mathrm{cm},$$
$$\Sigma_P = 5\times10^{11}\ \mathrm{cm/s},\quad \mathrm{k}_0 = 4.5\times10^{-7}\ \mathrm{cm}^{-1}.$$

Figure 3 illustrates the frequency spectrum of ionospheric pulsations to be observed on the ground, which suggests the excitation of very narrow-banded ionospheric pulsations in the ULF/ELF range. The separate frequencies are 2, 6, 14, 19 and 26 Hz in Figure 3.

4. Conclusion

[10] The coupling between the atmosphere and ionosphere (or even including the lithosphere) results in the formation of horizontal irregularities of ionospheric conductivity over a seismic region during the earthquake preparation phase. Interaction of these irregularities with the background electric field leads to an occurrence of periodic polarizing electric currents in the E layer dynamo region. The background electromagnetic noise is formed by both magnetospheric and atmospheric sources, such as lightning discharges. Polarizing electric currents are a coherent source of gyrotropic waves which propagate in the ionospheric conducting layer with finite thickness in the horizontal direction. Occurrence of the discrete spectrum of gyrotropic waves is caused by finite thickness of a layer in which they propagate. The dispersion relationship of discrete waves for a layer with Hall conductivity in a longitudinal magnetic field was considered by *Sorokin et al.* [2003]. The phase velocity of each discrete wave grows with an increase in frequency and with the number of wave by their dispersion. The electric current of waves induces magnetic pulsations on Earth's surface. In this paper we have considered the spectrum of waves generated by irregularities in the layer of Hall conductivity with finite thickness and in the layer of Pedersen conductivity. Then we have obtained a few spectral lines related with the thickness of the layer with Hall conductivity and also the absorption of waves concerned with Pedersen conductivity. If the source of waves is located in a horizontal direction with spatial scale, for example, of 100 km, the frequency spectrum of magnetic pulsations to be expected, has 6 lines in the ULF/ELF frequency 1–30 Hz as shown in Figure 3. Characteristics of the line spectrum are defined by both the parameters of the ionospheric irregularities and electrophysical parameters of the ionosphere. Basically, the amplitude of magnetic pulsation is defined by the intensity of ionosphere irregularities and its spatial structure and absorption of the wave. The frequency of spectral line maximums are determined by thickness of the conducting layer in this paper. The width of those spectral lines is defined by width of a spatial spectrum of ionosphere irregularities and the ratio between Pedersen and Hall conductivity. We expect that these line spectra, or narrow-banded ionospheric micropulsations would be observed in possible association with any earthquakes.

Appendix A

[11] We consider the ionosphere as two horizontal layers with various type of electric conductivity. The Hall conductivity is equal to zero in the upper layer, and the Pedersen conductivity is equal to zero in the lower layer as in Figure 2b. The length of gyrotropic waves is much larger than the thickness of conducting layers, therefore we can assume that the conductivities are constant inside of each layer. In the lower layer one finds from equation (2),

$$k^2 E_{1z} + i\frac{4\pi\omega}{c^2}\sigma_{H0}E_{1y} = -i\frac{4\pi\omega}{c^2}\sigma_{H0}f_H\left(\frac{d^2}{dz^2} - k^2\right)E_{1y} + i\frac{4\pi\omega}{c^2}\sigma_{H0}E_{1z} = 0 \quad \text{(A1)}$$

In the upper layer we have from equation (2),

$$k^2 E_{1z} - i\frac{4\pi\omega}{c^2}\sigma_{P0}E_{1z} = 0\left(\frac{d^2}{dz^2} - k^2\right)E_{1y} + i\frac{4\pi\omega}{c^2}\sigma_{P0}E_{1y} = - i\frac{4\pi\omega}{c^2}\sigma_{P0}f_P \quad \text{(A2)}$$

Then, we consider the equation for horizontal components of electric field in the lower layer. Eliminating E_{1z} in equation (A1), one finds,

$$\frac{d^2E_{1y}}{dz^2} - \left[k^2 - \left(\frac{4\pi\omega\sigma_{H0}}{c^2k}\right)^2\right]E_{1y} = -\left(\frac{4\pi\omega\sigma_{H0}}{c^2k}\right)^2 f_H. \quad \text{(A3)}$$

We assume that the Hall conductivity $\sigma_{H0}(z) = \sigma_0$ is constant inside the layer ($-l/2 < z < l/2$) in the Cartesian system of coordinates (as in Figure 2) and $\sigma_{H0}(z)$ is equal to zero outside this layer. Integrating equation (A3) over z in the vicinities of the upper and lower surfaces limiting the layer, we have:

$$\{E_{1y}\}_{l/2} = 0; \quad \left\{\frac{dE_{1y}}{dz}\right\}_{l/2} = 0; \quad \{E_{1y}\}_{-l/2} = 0; \quad \left\{\frac{dE_{1y}}{dz}\right\}_{-l/2} = 0. \quad \text{(A4)}$$

The braces are designated as the difference of function above and below the corresponding planes. The solution of equation (A3) inside this layer has a form:

$$E_{1y}(z) = C_1\exp(-qz) + C_2\exp(qz) + \left(\frac{\kappa_H}{q}\right)^2 \cdot \left\{1 - \cosh\left[q\left(z + \frac{1}{2}\right)\right]\right\}f_H \cdot q^2 = k^2 - \kappa_H^2, \kappa_H = 4\pi\omega\sigma_0/c^2k.$$

Defining C_1, C_2 constants and using equation (4) we obtain boundary conditions for electric field and their derivative on the lower layer with Hall conductivity:

$$E_{1y}\left(\frac{l}{2}\right) - \cosh(ql)E_{1y}\left(-\frac{l}{2}\right) - \frac{\sinh(ql)}{q}\frac{d}{dz}E_{1y}\left(-\frac{l}{2}\right) = \left(\frac{\kappa_H}{q}\right)^2 \cdot [1 - \cosh(ql)]f_H \frac{d}{dz}E_{1y}\left(\frac{l}{2}\right) - \cosh(ql)\frac{d}{dz}E_{1y}\left(-\frac{l}{2}\right) - q\sinh(ql)E_{1y}\left(-\frac{l}{2}\right) = \kappa_H^2\frac{\sinh(ql)}{q}f_H. \quad \text{(A5)}$$

Let us consider the upper layer of ionosphere with Pedersen conductivity. Assuming thickness of this layer tends to zero

$\sigma_{P0}(z) = \Sigma_P \delta(z - l/2)$ and integrating the second equation (A2) over z, we obtain:

$$\{E_{1y}\}_{l/2} = 0; \left\{\frac{dE_{1y}}{dz}\right\}_{l/2} + i\kappa_P E_{1y}\left(\frac{l}{2}\right) = -i\kappa_P f_P, \qquad (A6)$$

where Σ_P is the integrated Pedersen conductivity of the ionosphere, and $\delta(z)$ is a delta function. To add equations (A5) and (A6) we obtain the boundary conditions for electric field on the conducting ionosphere:

$$E_{1y}\left(\frac{l}{2}\right) - \xi_1 E_{1y}\left(-\frac{l}{2}\right) - \xi_2 \frac{d}{dz}E_{1y}\left(-\frac{l}{2}\right) = \varsigma_1 f_H \frac{d}{dz}E_{1y}\left(\frac{l}{2}\right) - \xi_3 \frac{d}{dz}E_{1y}\left(-\frac{l}{2}\right) - \xi_4 E_{1y}\left(-\frac{l}{2}\right) = -\varsigma_2 f_H - \varsigma_3 f_P \qquad (A7)$$

In the above formulas,

$$\xi_1 = \cosh(ql),\ \xi_2 = \frac{\sinh(ql)}{q},\ \xi_3 = \cosh(ql) - i\kappa_P \frac{\sinh(ql)}{q},$$

$$\xi_4 = q\sinh(ql) - i\kappa_P \cosh(ql), \varsigma_1 = \left(\frac{\kappa_H}{q}\right)^2 [1 - \cosh(ql)],$$

$$\varsigma_2 = \left(\frac{\kappa_H}{q}\right)^2 \{q\sinh(ql) + i\kappa_P[1 - \cosh(ql)]\},\ \varsigma_3 = i\kappa_P.$$

If we assume $\mathbf{E}_0 = 0$; $\Sigma_P = 0$ and we pass to a limit of $l \to 0$ and $\sigma_0 \to \infty$ under the condition of $\sigma_0^2 l = \int_{-\infty}^{\infty} \sigma_{H0}^2(z)\, dz =$ *const* in equation (A7) we obtain the boundary conditions for an infinitely thin conducting ionosphere which were considered by *Sorokin* [1988].

[12] **Acknowledgments.** We would like to thank NiCT for its support (research and development promotion scheme international joint funding).

[13] Zuyin Pu thanks Oleg Pokhotelov and another reviewer for their assistance in evaluating this paper.

References

Afonin, V. V., O. A. Molchanov, T. Kodama, M. Hayakawa, and O. A. Akentieva (1999), Statistical study of ionospheric plasma response to seismic activity: Search for reliable result from satellite observations, in *Atmospheric and Ionospheric Electromagnetic Phenomena Associated With Earthquakes*, edited by M. Hayakawa, pp. 597–603, Terra Sci., Tokyo.

Afraimovich, E. L., N. P. Perevalova, and A. V. Plotnikov (2002), Ionosphere response registration to the shock acoustic waves generating at rocket launches, *Geomagn. Aeron.*, *42*, 790–797.

Chmyrev, V. M., N. V. Isaev, O. N. Serebryakova, V. M. Sorokin, and Y. P. Sobolev (1997), Small-scale plasma inhomogeneities and correlated ULF emissions in the ionosphere over an earthquake region, *J. Atmos. Sol. Terr. Phys.*, *59*, 967–974.

Hayakawa, M., K. Ohta, S. Maekawa, T. Yamauchi, Y. Ida, T. Gotoh, N. Yonaiguchi, H. Sasaki, and T. Nakamura (2006), Electromagnetic precursors to the 2004 Mid Niigata Prefecture earthquake, *Phys. Chem. Earth*, *31*, 356–364.

Horie, T., T. Yamauchi, M. Yoshida, and M. Hayakawa (2007), The wave-like structures of ionospheric perturbation associated with Sumatra earthquake of 26 December 2004, as revealed from VLF observation in Japan of NWC signals, *J. Atmos. Sol. Terr. Phys.*, *69*, 1021–1028.

Maekawa, S., T. Horie, T. Yamauchi, T. Sawaya, M. Ishikawa, M. Hayakawa, and H. Sasaki (2006), A statistical study on the effect of earthquakes on the ionosphere, based on the subionospheric LF propagation data in Japan, *Ann. Geophys.*, *24*, 2219–2225.

Mareev, E. A., D. I. Iudin, and O. A. Molchanov (2002), Mosaic source of internal gravity waves associated with seismic activity, in *Seismo Electromagnetics: Lithosphere-Atmosphere-Ionosphere Coupling*, edited by M. Hayakawa and O. A. Molchanov, pp. 335–342, Terra Sci., Tokyo.

Miyaki, K., M. Hayakawa, and O. A. Molchanov (2002), The role of gravity waves in the lithosphere-ionosphere coupling, as revealed from the subionospheric LF propagation data, in *Seismo Electromagnetics: Lithosphere-Atmosphere-Ionosphere Coupling*, edited by M. Hayakawa and O. A. Molchanov, pp. 229–232, Terra Sci., Tokyo.

Molchanov, O. A., and M. Hayakawa (2008), *Seismo-Electromagnetics and Related Phenomena: History and Latest Results*, 189 pp., TERRAPUB, Tokyo.

Molchanov, O. A., M. Hayakawa, and K. Miyaki (2001), VLF/LF sounding of the lower ionosphere to study the role of atmospheric oscillations in the lithosphere-ionosphere coupling, *Adv. Polar Upper Atmos. Res.*, *15*, 146–158.

Rauscher, E. A., and W. I. Van Bise (1999), The relationship of extremely low frequency electromagnetic and magnetic fields associated with seismic and volcanic natural activity and artificial ionospheric disturbances, in *Atmospheric and Ionospheric Electromagnetic Phenomena Associated With Earthquakes*, edited by M. Hayakawa, pp. 459–487, Terra Sci., Tokyo.

Rozhnoi, A., M. S. Solovieva, O. A. Molchanov, and M. Hayakawa (2004), Middle latitude LF (40 kHz) phase variations associated with earthquakes for quiet and disturbed geomagnetic conditions, *Phys. Chem. Earth*, *29*, 589–598.

Rozhnoi, A., M. Solovieva, O. Molchanov, P. F. Biagi, and M. Hayakawa (2007), Observation evidences of atmospheric gravity waves induced by seismic activity from analysis of subionospheric LF signal spectra, *Nat. Hazards Earth Syst. Sci.*, *7*, 625–628.

Sorokin, V. M. (1988), Wave processes in the ionosphere related to the geomagnetic field, Izvestia Vuzov, *Radiophysics*, *31*, 1169.

Sorokin, V. M. (2007), Plasma and electromagnetic effects in the ionosphere related to the dynamic of charged aerosols in the lower atmosphere, *Russ. J. Phys. Chem.*, *1*(2), 138–170.

Sorokin, V. M., and G. V. Fedorovich (1982), Propagation of short-period waves in ionosphere, *Radiophysics*, *25*, 495–501.

Sorokin, V. M., and O. A. Pokhotelov (2005), Gyrotropic waves in the mid-latitude ionosphere, *J. Atmos. Sol. Terr. Phys.*, *67*, 921–930.

Sorokin, V. M., V. M. Chmyrev, and N. V. Isaev (1998), A generation model of small-scale geomagnetic field-aligned plasma inhomogeneities in the ionosphere, *J. Atmos. Sol. Terr. Phys.*, *60*, 1331–1342.

Sorokin, V. M., V. M. Chmyrev, and A. K. Yaschenko (2003), Ionospheric generation mechanism of geomagnetic pulsations observed on Earth's surface before earthquake, *J. Atmos. Sol. Terr. Phys.*, *65*, 21–29.

M. Hayakawa, Research Station on Seismo Electromagnetics, Department of Electronic Engineering, University of Electro-Communications, 1-5-1 Chofugaoka, Chofu Tokyo 182-8585, Japan. (hayakawa@whistler.ee.uec.ap.jp)

V. M. Sorokin, Ionosphere and Radio Wave Propagation, Institute of Terrestrial Magnetism, Troitsk, Moscow 142190, Russia.