

Comparative characteristics of stellar and sunspot spectra

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A comparative analysis of the spectra of dwarf stars and a sunspot is made. It is shown that the spectral class of a sunspot depends on the index used: A sunspot has the class M0 based on the continuous spectrum, about K5 based on weak lines, and G8–K0 based on strong lines. From this it follows that the atmosphere of a sunspot cannot be likened to the average standard atmosphere of a star of any spectral class. This may be connected with the specific role of magnetic fields, the inhomogeneous structure of the atmosphere, and horizontal energy transfer in a sunspot, and with the force of gravity, different from that of a star with the same T_{eff} .

1. INTRODUCTION

Numerous investigations of recent years have shown that a sunspot is a second stable state of solar material.¹ Sunspots of the most varied sizes hardly differ in brightness, temperature, density, magnetic field, and other physical parameters. We do not observe any intermediate states between the undisturbed photosphere and a sunspot. Thus, the contrast of a sunspot in integral light is 0.15 and the scatter of the actually observed values does not exceed 10%. Generally speaking, it is not entirely clear why the combination of various physical mechanisms leading to sunspot formation yields just such a stable state. This surprising stability hinders the understanding of the nature of a sunspot, since we have no experimental evidence of how the properties of the plasma vary upon small variations of the main model parameters.

Plasma in such a state can only be observed in stars of a close spectral class. Therefore, it is natural to attempt to compare the emission characteristics of the plasma of a sunspot with the characteristics of the plasma of a star close in temperature. If one could find a typical star whose spectrum corresponds exactly to the spectrum of a sunspot, one would expect that the model characteristics of the atmosphere of such a star and a sunspot would coincide and the mechanisms forming these atmospheres were close. In this case it is essential that stars form a series of objects continuous in their characteristics, and so one can hope both to choose a star with the closest spectral class and to trace the variation of the optical characteristics as the physical conditions vary.

Since the main parameter of a stellar atmosphere is the heat flux coming from below, while the role of the magnetic field in the formation of a sunspot consists mainly in the change in this flux in comparison with the photosphere, such a star probably can be chosen. There are circumstances, however, making the question of the possibility of likening a sunspot to the atmosphere of some concrete star nontrivial. This is connected, first of all, with the fact that the role of the magnetic field comes down to not only a change in the amount of heat flux arriving at the sunspot but also a change in the principal mechanisms of energy transfer. Instead of convective transfer, which plays the decisive role in subphotospheric layers, in a sunspot energy is transferred by radiation and wave processes.² The role of waves is enhanced still more in the chromosphere and corona above a sunspot. Moreover, the magnetic field can considerably alter the equiva-

lent widths of spectral lines. Second, a sunspot consists of a plasma formation submerged in the atmosphere of a star of a different spectral class. Since for main-sequence stars each value of the effective temperature corresponds to its standard value of the acceleration of gravity at the stellar surface, for a sunspot we encounter a specific case when an atmosphere with a far lower T_{eff} has the same g as a star of class G2 V. This means that, under the condition of hydrostatic equilibrium, the height variation of the temperature will be different from that in a star with the same T_{eff} . In addition, as shown in Refs. 3–5, even for sunspots of rather large size the lateral transfer of radiation energy can also greatly alter the temperature model of the sunspot. These changes in the model must result in the fact that the spectral class of the spot determined from different indices (the continuous spectrum, lines of different strengths, etc.) will differ somewhat.

Finally, third, we must keep in mind that the conditions of observations of sunspots and stars are entirely different. For a star we have the total integral flux from the entire disk. But a sunspot is usually observed at the center of the disk and, as a rule, sections of the sunspot umbra free of nonuniformities are chosen. Therefore, strictly speaking, for comparison with stars one ought to use some averaged sunspot spectrum with allowance for center-to-limb variations and the presence of nonuniformities.

Unfortunately, the task of the direct comparison of sunspot and stellar spectra encounters numerous difficulties. On the one hand, this is connected with the fact that the sunspot spectrum is distorted with considerable stray light from the surrounding photosphere. On the other hand, there are still rather few observations of the spectra of stars of late classes with a sufficiently high dispersion. All the same, we attempted to make some progress in this direction.

Estimates of the spectral class of a sunspot have been made for a very long time now. We recall only some of them. Hale long ago drew attention to the fact that the spectrum of a sunspot corresponds to the spectrum of a star of considerably later spectral class than the sun. In 1930 Pettit and Nicholson⁶ determined the value of the spectral class of a sunspot as G7. In 1946 van Dijke⁷ showed that the spectrum of a sunspot is close to the spectrum of the K0 V star 70 Oph. This latter value appeared in all handbooks for a long time (see, e.g., Allen's

1973 handbook). Mel'nikov and Zhuravlev named classes from G8 to K2 in various papers.⁸⁻¹⁰ Of recent estimates we must cite Ref. 11, where classes K0-K5 are indicated, and Ref. 12, specially devoted to estimating certain stellar characteristics of a sunspot. Unfortunately, in that paper the spectral class (K7-M1) was determined not by a direct comparison of spectra but by comparing the temperatures in models of a sunspot and of late stars.

2. CONTINUOUS SPECTRUM

First we make some general estimates.

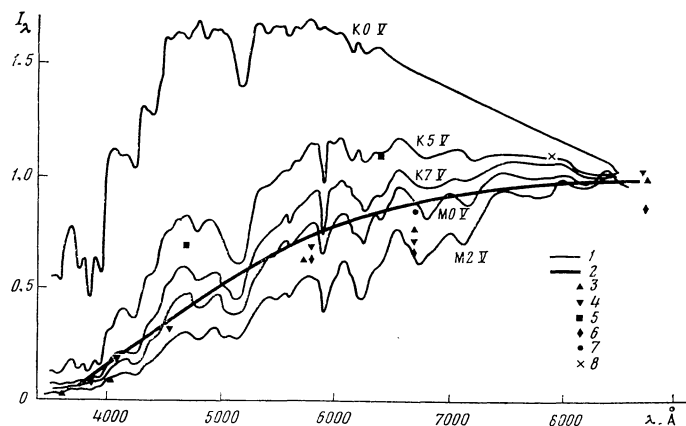
In present-day models¹³⁻¹⁵ the effective temperature of a sunspot is taken as 3650-3950°K. Using the table of effective temperatures in Lang's handbook,¹⁶ we obtain the spectral class M0-M1.

The B-V color index can be calculated from the curve of the wavelength dependence of the continuous spectrum. Using the averaged spectrum from Ref. 17, we obtain B-V = 1.22-1.44. Turning again to the handbook Ref. 16, we obtain the spectral class M0-M2 and the effective stellar magnitude $M_V = 9.1$.

These estimates show that the spectrum of a sunspot should be compared with the spectra of stars considerably later than K0. In this case the most reliable results can be obtained through a direct comparison of the energy distributions in the spectra of sunspots and stars of different classes. And we made such a comparison using stellar spectra from the catalog Ref. 18. The values of monochromatic intensities F_{λ}^* averaged over an interval $\Delta\lambda = 50 \text{ \AA}$, are given in the catalog. To characterize the shape of the spectrum, the normalized values of the monochromatic intensity I_{λ}^* , obtained by dividing the values of F_{λ}^* by the intensity $F_{0.88}^*$ at $\lambda^* = 0.88 \text{ \mu m}$, $I_{\lambda}^* = F_{\lambda}^*/F_{0.88}^*$, were used.

To characterize the spectrum of a sunspot we chose: a) the average curve from Ref. 17; b) the wide-band observations of Refs. 19-24. Since the relative monochromatic intensities φ_{λ} of a sunspot in fractions of the intensity of the adjacent undisturbed photosphere are given in Refs. 17 and 19-24, we calculated the normalized intensity for a spot:

$$I_{\lambda}^s = \frac{\varphi_{\lambda}}{\varphi_{0.88}} \cdot \frac{F_{\lambda}^{\odot}}{F_{0.88}^{\odot}}. \quad (1)$$



Here F_{λ}^{\odot} and $F_{0.88}^{\odot}$ are the intensities of emission of the center of the sun at the arbitrary wavelength and at $\lambda = 0.88 \text{ \mu m}$, respectively. The value of the relative intensity of a sunspot at $\lambda = 0.88 \text{ \mu m}$ is¹⁹ 0.2, and this was taken as the reference value of $\varphi_{0.88}$.

The formula (1) means that we are comparing the stellar spectrum with the spectrum of a sunspot located at the center of the disk. We note that φ_{λ} depends weakly on the position of the sunspot on the disk, since the center-to-limb variations for a sunspot and for the undisturbed photosphere are close.

In Fig. 1 we present the variations of I_{λ}^* from Ref. 18, experimental data on I_{λ}^s from Refs. 19-24, and the average I_{λ}^s curve from Ref. 17. The narrow-band spectral observations are not plotted in Fig. 1, since they are not suitable for comparison with low-dispersion stellar spectra.

It is seen from Fig. 1 that the average curve from Ref. 17 agrees best with the spectrum of a star of class M0. The original wide-band measurements fall mainly in the region somewhat below the curve for the class M0 V but above that for the class M2 V. Thus, we arrive at the conclusion that the spectral class of a sunspot based on the continuous spectrum is M0 V-M1 V.

3. WEAK FRAUNHOFER LINES

The spectral class of a sunspot based on weak spectral lines ($W_{\odot} \leq 100 \text{ m\AA}$) could be established using curves of growth. However, this method is rather laborious, requires knowledge of a number of constants of the spectral lines, and depends strongly on the accuracy of the photometric scales. In addition, a curve of growth is constructed separately for the lines of each element. More convenient is a method which can be called the method of enhancement diagrams.

Suppose we have a sufficiently large statistical set of data on the equivalent widths of lines for stars, the sun, and sunspots. We shall use the ratios of the equivalent widths of lines in the spectra of stars (W^*) and sunspots (W_s) to the equivalent widths of the same lines in the spectrum of the solar photosphere:

$$u = \frac{W^*}{W_{\odot}}; \quad u_s = \frac{W_s}{W_{\odot}}. \quad (2)$$

FIG. 1. Normalized intensity in the continuous spectrum as a function of wavelength for K0 V-M2 V stars (1); 2) average curve for a sunspot from Ref. 17 and individual wide-band measurements; 3) Ref. 19; 4) Ref. 20; 5) Ref. 21; 6) Ref. 22; 7) Ref. 23; 8) Ref. 24.

We shall call u and u_s the measure of line enhancement. Since the equivalent widths of the majority of photospheric lines grow with a decrease in temperature, while we shall investigate stars later than the sun, the quantities u and u_s are usually larger than one.

We now construct the enhancement diagram as follows. We lay out $\log u_j$ along the abscissa axis, where u_j is the measure of enhancement in the star whose spectral class must be determined. Then the quantity $\log u_j/u_i$ is laid out along the ordinate axis, where u_i is the measure of enhancement in a certain standard star with a known spectral class. It is easy to see that if the standard star j has a later spectral class than the investigated star i , we obtain a growing dependence of $\log u_j/u_i$ on $\log u_i$. Otherwise it will be a declining dependence. If the spectral classes of the stars j and i coincide, one would expect that the experimental points will concentrate around the straight line $\log u_j/u_i = 0$ to within the scatter due to observational errors and the difference in the photometric scales. In this case a dependence on $\log u_i$ should be absent or weakly expressed in the case when the atmospheres of these stars are not fully identical.

The rough quantitative scheme of the method can be given as follows. In the case of LTE, for very weak lines of an element which is in the r -th stage of ionization with an excitation potential χ , the measure of enhancement can be expressed as

$$\lg u_i \approx \lg \left(\frac{n_r}{\sum n_k} \right)_i - \lg \left(\frac{n_r}{\sum n_k} \right)_\odot - \Delta\theta_{i\odot}\chi, \quad (3)$$

where the relative fractions of ions in the r -th stage of ionization, $n_r/\sum n_k$, is estimated from the Saha formula:

$$\Delta\theta_{ij} = 5040 \left(\frac{1}{T_i} - \frac{1}{T_j} \right). \quad (4)$$

From Eq. (3) it is easy to obtain the connection

$$\lg \frac{u_j}{u_i} = a + \frac{\Delta\theta_{ji}}{\Delta\theta_{i\odot}} \lg u_i, \quad (5)$$

where

$$a = \lg \left(\frac{n_r}{\sum n_k} \right)_j - \lg \left(\frac{n_r}{\sum n_k} \right)_i + \frac{\Delta\theta_{ji}}{\Delta\theta_{i\odot}} \left[\lg \left(\frac{n_r}{\sum n_k} \right)_\odot - \lg \left(\frac{n_r}{\sum n_k} \right)_i \right]. \quad (6)$$

It is easy to see that for $i = j$, i.e., when the spectral classes of the standard and investigated stars coincide, the right and left sides of Eq. (5) are reduced to zero. When the spectral classes do not coincide, one should observe an approximately linear dependence of the type (5). If star j is earlier than star i , its temperature is higher, $\Delta\theta_{ji}$ is negative, and the dependence is declining, and for the opposite relation of the spectral classes the dependence will be a growing one. It is important to note that the angular coefficient of this dependence is determined only by the ratio of the model parameters and does not depend on any atomic constants and line parameters. Moreover, if the range of spectral classes within which the investigated star lies is known preliminarily from some considerations, then the quantity a depends weakly on the element used. Thus, in the range of spectral classes K0-K5 for all the elements used by us the quantity

a lies in the range of 0.20 ± 0.10 , which hardly exceeds the experimental scatter (see Fig. 2a below). This enables one to refine the spectral class of a star with allowance for the complete set of spectral lines of different elements.

The possibilities of the method and its accuracy are illustrated by Fig. 2a and b. A comparison of the measures of enhancement for two stars of the same spectral class is shown in Fig. 2a. The star HD 192310, used as the standard, has the spectral class¹⁾ K2 V with weak Ca II emission,²⁵ and the K2 V star HD 190404 is without Ca II emission.²⁶ It is seen that a dependence is practically absent, the weak tendency toward growth lies fully within the limits of the statistical scatter, and the lines in HD 190440 are somewhat weaker than those in HD 192310, which may be determined simply by differences in the photometric procedures in Refs. 25 and 26.

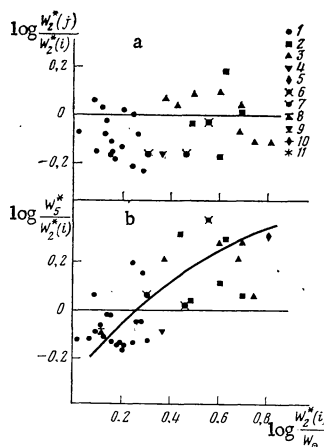


FIG. 2. Ratios of equivalent line widths in two stars of class K2 V (a) and of classes K2 V and K5 V (b) as a function of the measure of line enhancement in a K2 V star. Notation: 1) Fe I; 2) Ti I; 3) V I; 4) Ni I; 5) Zr I; 6) Sc I; 7) Na I; 8) Sc II; 9) Ca I; 10) Si I; 11) Fe II.

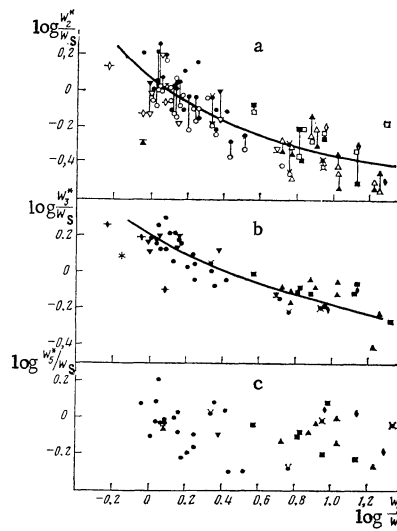


FIG. 3. Ratios of equivalent widths in stars of classes K2 V (a), K3 V (b), and K5 V (c) and a sunspot as a function of the measure of line enhancement in the sunspot. Notation same as in Fig. 2. In Fig. 3a the dark symbols pertain to the star HD 192310 and the light ones to the star HD 190404.

TABLE I

Class	Star	HD	Fe I 4325,8	Fe I 4383,6	Fe I 4404,8	Ca I 4226,7	References
G5 V		178428				1.45	[34]
G8 V	ε Boo	134156	1.00	1.90		2.04	[34]
		61 UMa	1.72	1.87		1.76	[34]
K0 V	δ Pav	190248	0.82	1.05	1.37		[35]
		70 Oph A	0.92	1.24	1.06		[35]
			2.42	2.61	2.13	2.74	[34]
			2.27		0.94		[36]
			166	1.02	1.63	1.81	2.15
K2 V	σ Dra	10780	1.86	1.97	1.50	2.11	[34]
			1.09	2.05	1.24		[35]
			1.64	2.13	1.50	2.00	[34]
			1.06	1.21	1.11		[35]
K3 V	54 Psc	3651	0.99	1.48	1.08		[35]
		22049	1.09	2.10	1.16		[35]
K5 V	ε Eri	192310		3.30	2.40	3.36	[25]
		219134	2.24	2.84	2.14	4.05	[34]
K5 V	61 Cyg A	201091	3.63	3.07	2.50	6.05	[34]
				2.70			[37]
K7 V	36 Oph C	156026		4.00	2.70	10.1	[36]
		61 Cyg B	201092		4.50	3.00	10.0
dMO		88230	2.19	2.30	2.15	6.67	[25]
Sunspot			0.89	1.77	1.19	1.85	[37]
			0.94	2.15	1.33	2.04	[38] see text [11] without allowance for scattered light [11]with allowance for scattered light; see text
						2.4±0.3	[39]

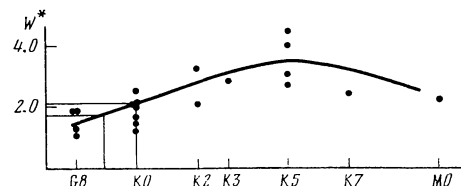


FIG. 4. Equivalent width of the λ 4383,6 Å Fe I line as a function of spectral class. Two values of the equivalent width in a sunspot from Ref. 11, with and without allowance for scattered light, are indicated on the ordinate axis.

The enhancement diagram for two stars clearly differing in spectral class (although not very greatly), HD 192310 K2 V and HD 156026 K5 V (Ref. 25), is shown in Fig. 2a. As expected, the dependence of $\log(W_5^*/W_2^*)$ on $\log(W_2^*/W_\odot)$ is clearly noted.

Enhancement diagrams in which the investigated "star" is a sunspot are shown in Fig. 3a, b, and c. The data on the equivalent widths W_s and W_\odot are taken from Ref. 27. As the standard stars we took HD 192310 K2 V and HD 190404, HD (Refs. 25 and 26) (Fig. 3a; points pertaining to the same spectral line are connected; darkened symbols pertain to the star HD 192310 and light ones to the star HD 190404), HD 219134 K3 V (Ref. 26) (Fig. 3b), and HD 156026 K5 V (Ref. 25) (Fig. 3c). A clear decline in $\log u_j/u_i$ is seen in Fig. 3a and b, and this shows that the sunspot has a later atmosphere than K2 and K3. In Fig. 3c there is no clear dependence. The scatter does not exceed that which we saw in Fig. 2a, where two stars of the same spectral class were compared.

Thus, on the basis of weak Fraunhofer lines we conclude that the spectral class is close to K5. A comparison of other observations of lines in sunspots^{13,28-32} and the spectra of the above-mentioned stars, as well as 70 Oph K0 V (Ref. 33) and 61 Cyg A K5 V (Ref. 25), leads to the same conclusion, at least for large sunspots.

4. STRONG FRAUNHOFER LINES

For strong Fraunhofer lines we could not apply a statistical analysis using enhancement diagrams owing to the absence of a sufficient amount of data. Data on measurements of four lines with equivalent widths of about 1 Å or more in the solar photosphere are presented in Table I.

Values of equivalent line widths for the star HD 88230 of class dM0, calculated by us from the high-dispersion atlas Ref. 38 under the assumption of a triangular line profile, are included in Table I. It should be mentioned that uncertainty in drawing the continuous spectrum can lead to a considerable underestimate of equivalent widths. Thus, the decline in the values of the equivalent widths after K5 is possibly connected with this effect.

Data on equivalent line widths obtained in Ref. 11 are presented in Table I for a sunspot. To estimate the upper limit for scattered light we assumed that H γ in a sunspot is fully determined by scattered light. Using the value of 0.955 Å from Ref. 11 for the equivalent width of this line, we find that the upper limit of the amount of scattered light is 50% in fractions of the intensity of the continuous spectrum of the umbra. Data corrected with this value of the scattered light are also given in Table I.

It follows from the table that the equivalent widths of these lines in a sunspot are close to those observed in stars of spectral classes G8-K0. Figure 4 illustrates the dependence of the equivalent width of the λ 4383,6 Å Fe I line on the spectral class of the star. The value of the equivalent width of this line in a sunspot corrected for scattered light (2.15 Å) corresponds to the spectral class K0, while the uncorrected value (1.77 Å) corresponds to the class G9.

A comparison of the profiles of the Mg I b₁, Na I D₂, and H α lines^{40,41} with the corresponding profiles in stars (Refs. 25 and 26) shows that in these layers the atmosphere

above a sunspot roughly corresponds to the class K0-K2. At present it is hard to say anything definite about lines arising even higher in the atmosphere of a sunspot (e.g., λ 10830 Å He I or Ca II H and K), since the profiles of these lines depend strongly on the individual properties both of the sunspot and the star.

5. DISCUSSION OF RESULTS

Thus, the determinations of the spectral class of a sunspot from different characteristics of its spectrum do not coincide. The spectral class is determined most reliably from the continuous spectrum. We can take the new value of M0 for the class of a sunspot as definite. At the same time, based on weak lines the spectral class is K5 and based on strong lines it is G8-K0. This means that the atmosphere of a sunspot cannot be fitted into the general sequence of standard stellar atmospheres. As indicated in the introduction, the differences may be due both to horizontal energy transfer and to the value of the force of gravity g , not usual for classes K0 V-M0 V. It can be expected that the actual solar value of g for a sunspot having T_{eff} corresponding to the class M0 shifts the sunspot toward supergiants on the Hertzsprung-Russell diagram.

Additional causes of the mismatch of the models may be connected with the fact that in both a sunspot and stars there are considerable inhomogeneities, which lie at the limit of resolution in sunspots and, naturally, are entirely unresolved in stars. Their roles may be different. In this case one cannot talk at all about any average models of the atmosphere, and a comparison must be made with allowance for the inhomogeneities. For sunspots the allowance for fine structure lets one reconcile observations of the continuous spectrum and of equivalent line widths.⁴²

One must keep in mind that the role of the magnetic field in the formation of stellar atmospheres is complicated and diverse. The point is that the strong magnetic field of ~ 3000 G undoubtedly plays a definite role in the formation of the atmosphere of a sunspot. Such strong magnetic fields are also detected in stars of late classes, although they evidently do not encompass the entire stellar atmosphere but are concentrated in individual sections of the starspot type. Obviously, in this case the roles of the magnetic field in the formation of the spectra of a sunspot and a star must be different on the whole. It is possible that the difficulties with the selection of a single spectral class for a sunspot are analogous to the difficulties arising in the interpretation of the spectra of magnetic stars of the A_p and A_m classes.^{43,44}

In any case, the observed difference indicates that either the vertical stratification or the organization of the fine structure in a sunspot are considerably different from those in stars.

It should be mentioned that the material used in the paper is very heterogeneous, and for a more precise

analysis it would be desirable to conduct special observations.

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Translated by Edward U. Oldham