Depression of the ULF geomagnetic pulsation related to ionospheric irregularities

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Abstract

We consider a depression in intensity of ULF magnetic pulsations, which is observed on the ground surface due to appearance of the irregularities in the ionosphere. It is supposed that oblique Alfven waves in the ULF frequency range are downgoing from the magnetosphere and the horizontal irregularities of ionospheric conductivity are created by upgoing atmospheric gravity waves from seismic source. Unlike the companion paper by Molchanov *et al.* (2003), we used a simple model of the ionospheric layer but took into consideration the lateral inhomogeneity of the perturbation region in the ionosphere. It is shown that ULF intensity could be essentially decreased for frequencies f = 0.001-0.1 Hz at nighttime but the change is negligible at daytime in coincidence with observational results.

Key words *ULF – ionosphere – Alfven – seismicity*

1. Introduction

While influence of plasma irregularities is important for radio-wave propagation through the ionosphere, the same influence for ULF waves is usually neglected. In the conventional approach to many geophysical problems related to ULF magnetic pulsations (magnetospheric diagnostics, wave-particle interaction inside radiation belts and so on), the ionosphere is approximated by a layer with homogeneous conductivity, which is connected with the regular conductivity profile integrated on height. However there are some indications that such a simple model is not valid even for the ULF frequency range taking into account the existence of ionospheric perturbations in reality. Alperovich *et al.* (2002) using both theoretical computations and laboratory experiments showed that rather small (~ 10-30%) variations in plasma density can produce a several times increase in «effective» conductivity, included in consideration of ULF wave characteristics. Molchanov *et al.* (2003) reported the results of ULF magnetic field observations (0.003-5.0 Hz) at Kamchatka region during a long period of seismic activation. They found a remarkable and statistically reliable ULF intensity depression several days before strong seismic shocks. The effect was especially clear at nighttime and for the filter channels 0.01-0.1 Hz and it was absent at daytime. They interpreted the effect in as-

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sumption of atmospheric gravity wave intensification induced by changes in temperature and pressure near the ground due to gas and water release in a course of earthquake preparation. Mareev *et al.* (2002) considered gravity waves intensification in the ionosphere. An appearance of gravity waves in the ionosphere leads to depression of downgoing from magnetosphere ULF waves due to loss of coherency along the wave front (like scattering) and due to a change in effective ionospheric conductivity. Here we are going to investigate the latter process theoretically. Some results on this subject are obtained in a companion paper by Molchanov *et al.* (2003) in assumption of vertical stratification of the ionosphere. Here we consider the same effect in assumption of lateral inhomogeneity but for thin ionospheric layer.

2. Electromagnetic ULF field in horizontally inhomogeneous ionosphere

We use a simple model, which includes the following:

i) Source of the magnetic field ULF pulsations is situated in the magnetosphere and it generates downgoing Alfven waves. They propagate in a homogeneous magnetospheric medium above ionosphere along z-axis, which is coincident with the direction of the external magnetic field. The frequency of the waves $\omega << \omega_{ci}$, where ω_{ci} is ion cyclotron frequency, therefore they propagate with characteristic Alfven wave velocity C_A and their vertical wave number $k_z = k_A = \omega / C_A$. In addition we suppose their horizontal wave number $k >> k_A$, another words we consider oblique Alfven waves.

ii) Ionosphere is a thin layer at $z = 0$ with integrated Pedersen and Hall conductivities $\Sigma_P \cdot (x, t)$, $\Sigma_H (x, t)$ respectively, which are time-dependent and inhomogeneous along the horizontal x-axis (see fig. 1). For simplicity, we neglect input due to ionosphere thickness Δh, because of $k_A \Delta h << 1$ and present

$$\Sigma_{P,H}(x,t) = \Sigma_{P,H_0} + \Delta\Sigma_{P,H}(x,t). \qquad (2.1)$$

iii) Atmosphere below ionospheric layer is nonconductive and current-free, but the ground medium is completely conductive and tangential electric field disappears at the ground surface $z = h$.

iv) For simplicity, we assume independence of the all field components on y-coordinate, *i.e.* $\partial / \partial y = 0$, and suppose large conductivity along z-axis for magnetosphere and ionosphere that leads to disappearance of electric field component $E_z \sim 0$.

Our model is about the same as was discussed in many other papers (*e.g.,* Lyatsky and Maltsev, 1983). The only difference is assumption on lateral ionospheric inhomogeneity.

Alfven wave can reflect from the ionospheric layer and transform in the isotropic mode wave (so-called fast magneto-sonic wave) inside ionosphere. Then both waves penetrate into the Earth-ionospheric cavity. As usual we consider Fourier expansion

$$E_r(x,z,t) = \int_{-\infty}^{\infty} \frac{dk}{2\pi} \int_{-\infty}^{\infty} \frac{d\omega}{2\pi} E_r(k,z,\omega)\, e^{(ikx - i\omega t)} \qquad (2.2)$$

where index $r = x, y,$ and the same for magnetic components b_r. In our model for Alfven wave in the magnetosphere only E_x and b_y exist and

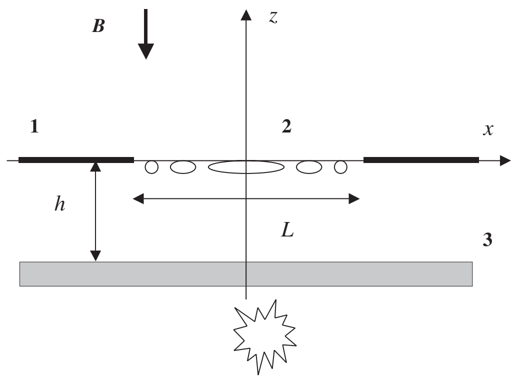

Fig. 1. The model used to calculate the depression of the ULF geomagnetic pulsations: 1 – conducting ionosphere; 2 – ionospheric inhomogeneities; 3 – Earth surface. h is the height of the lower boundary of the ionosphere; L is the spatial scale of the seismic region.

these spectral component are described by following equations:

$$\frac{\partial^2 E_x(x,z,\omega)}{\partial z^2} + k_A^2 E_x(k,z,\omega) = 0$$
$$b_y = \frac{1}{i\omega}\frac{\partial E_x}{\partial z}, \quad k_A = \frac{\omega}{C_A}. \tag{2.3}$$

In opposite only component E_y, b_x, b_z keep in the isotropic wave

$$\frac{\partial^2 E_y(x,z,\omega)}{\partial z^2} + k_i^2 E_y(k,z,\omega) = 0$$
$$b_x = -\frac{1}{i\omega}\frac{\partial E_y}{\partial z}, \quad b_z = \frac{k}{k_0} E_y \tag{2.4}$$
$$k_i^2 = k_A^2 - k^2, \quad k_0 = \omega/c.$$

As concerned the situation below ionosphere (at the atmosphere) we have well-known relationships

$$\frac{\partial^2 E_r(x,z,\omega)}{\partial z^2} + (k_0^2 - k^2)\, E_r(k,z,\omega) = 0$$
$$b_x = -\frac{1}{ik_0}\frac{\partial E_y}{\partial z} = -\frac{k_{z0}}{k_0} E_y \tag{2.5}$$
$$b_y = \frac{ic}{\omega}\left(\frac{\partial E_x}{\partial z} - \frac{\partial E_z}{\partial x}\right) = \frac{k_0}{k_{z0}} E_x$$

where $k_0^2 << k^2$ and $k_{z0}^2 = k_0^2 - k^2 \approx -k^2$.

Then after matching of the magnetospheric and atmospheric fields in the ionosphere layer we obtain integral equation for the fields inside ionosphere (see Appendix for details)

$$\widehat{K}(k,\omega)\boldsymbol{E}(k,0,\omega) = 2k_A \boldsymbol{E}_*(k) - \frac{4\pi\omega}{c^2}\cdot$$
$$\cdot \int_{-\infty}^{\infty} dk' \Delta\widehat{\Sigma}(k-k')\boldsymbol{E}(k',0,\omega) \tag{2.6}$$

where

$$\boldsymbol{E} = \begin{bmatrix} E_x \\ E_y \end{bmatrix}, \ \boldsymbol{E}_* = \begin{bmatrix} E_* \\ 0 \end{bmatrix}, \ \widehat{K} = \begin{bmatrix} k_1 & -k_H \\ k_H & k_2 \end{bmatrix}$$
$$\Delta\widehat{\Sigma} = \begin{bmatrix} \Delta\Sigma_P & -\Delta\Sigma_H \\ \Delta\Sigma_H & \Delta\Sigma_P \end{bmatrix}, \ k_1 = k_P + k_i + ik\coth(kh)$$
$$k_2 = k_P + k_A, \quad k_{P,H} = 4\pi\Sigma_{P,H}\,\omega/c^2$$
$$k_A = 4\pi\Sigma_A\,\omega/c^2, \ \Sigma_A = c^2/4\pi C_A.$$

Its connection with ULF magnetic field at the ground is also analyzed in Appendix, where shown $b_x(\omega, h) >> b_y(\omega, h)$ and

$$b_x(k,h,\omega) = -i\left\{k/[\omega\sinh(kh)]\right\} E_y(k,0,\omega). \tag{2.7}$$

3. Change in the ULF spectrum on the ground induced by ionospheric perturbations

Using conventional approach of perturbation theory we transform (2.6) as following:

$$\boldsymbol{E}(k,0,\omega) = 2k_A\widehat{K}^{-1}\boldsymbol{E}_* - \frac{4k_A\omega}{c^2}\widehat{K}^{-1}\cdot$$
$$\cdot \int_{-\infty}^{\infty} dk' \Delta\widehat{\Sigma}(k-k')\widehat{K}^{-1}(k',\omega)\boldsymbol{E}_*. \tag{3.1}$$

where inverse tensor $\widehat{K}^{-1}$ is easy to find

$$\widehat{K}^{-1} = \frac{1}{\kappa}\begin{bmatrix} k_1 & k_H \\ -k_H & k_2 \end{bmatrix}, \ \kappa(k) = k_1(k)\,k_2 + k_H^2.$$

After substitution in (2.7) we have

$$b_x(k,h,\omega) = 2i\frac{ckk_A k_H}{\omega\kappa(k,\omega)\sinh(kh)} E_*(\omega,k) +$$
$$-i\frac{4kk_A}{c\kappa(k,\omega)\sinh(kh)}\int_{-\infty}^{\infty} dk' \{\Delta\Sigma_P(k-k')\cdot$$
$$\cdot M_1(k',\omega) - \Delta\Sigma_H(k-k')M_2(k',\omega)\}\, E_*(\omega,k') \tag{3.2}$$

where

$$M_1(k) = \frac{1}{\kappa(k)}k_H[k_1(k) + k_2]$$
$$M_2(k) = \frac{1}{\kappa(k)}k_H[k_1(k)\,k_2 - k_H^2]\cdot$$

ULF magnetic field at the ground we find as the inverse Fourier transform of (3.2)

$$b_x(x,h,\omega) = b_{x0}(x,h,\omega) - b_{x1}(x,h,\omega)$$

$$b_{x0}(x,h,\omega)=\frac{ick_A k_H}{\pi\omega}\int_{-\infty}^{\infty}dke^{ikx}\frac{kE_*(\omega,k)}{\kappa(k,\omega)\sinh(kh)}$$
$$b_{x1}(x,h,\omega)=\frac{2ik_A}{\pi c}\int_{-\infty}^{\infty}dke^{ikx}\frac{k}{\kappa(k,\omega)\sinh(kh)}\times$$
$$\times\int_{-\infty}^{\infty}dk'\{\Delta\Sigma_P(k-k')M_1(k',\omega)-\Delta\Sigma_H(k-k')\cdot$$
$$\cdot M_2(k',\omega)\}E_*(\omega,k'). \quad (3.3)$$

In (3.3) the second term b_{x1} is due to presence of perturbations in the ionosphere and could be compared with b_{x0}. As example for solitary initial wave $E_*(x)=E_*\exp(ik_x x)$ and $E_*(k)=2\pi E_*\delta(k+-k_x)$. We obtain

$$b_{x0}(x,h,\omega)=2E_*\frac{ick_A k_H k_x}{\omega\kappa(k_x)\sinh(k_x h)}e^{ik_x x}$$
$$b_{x1}(x,h,\omega)=4E_*\frac{ik_A}{c}\int_{-\infty}^{\infty}dke^{ikx}\frac{k}{\kappa(k)\sinh(kh)}\times$$
$$\times\{\Delta\Sigma_P(k-k_x)M_1(k_x)-\Delta\Sigma_H(k-k_x)M_2(k_x)\} \quad (3.4)$$

where $k_x=2\pi/\lambda_x$ and λ_x is horizontal scale of downgoing Alfven wave. Let us consider influence a moving density variation in the ionosphere (*e.g.,* gravity waves). Alperovich *et al.* (2002) showed that such a variation leads to mainly perturbation of Pedersen conductivity. So in (3.4) we leave only the first term in the integrand and present the perturbation as following:

$$\Delta\Sigma_P(x)=2\Delta\Sigma_{P0}\left[\cos\frac{k_g x-\varphi}{2}\right]^2\exp\left(-\frac{x^2}{4L^2}\right)$$
$$\Delta\Sigma_P(k)=2\sqrt{\pi}\,L\Delta\Sigma_{P0}\left\{e^{-k^2L^2}+\frac{1}{2}\cdot\right.$$
$$\left.\cdot\left[e^{-(k_g-k)^2L^2-\varphi}+e^{-(k_g+k)^2L^2+\varphi}\right]\right\} \quad (3.5)$$

where $\Delta\Sigma_{P0}$ is averaged amplitude of the perturbation induced by gravity wave, φ is random phase of the gravity wave, L is spatial scale of the seismic region. Substituting (3.5) in (3.4) we obtain the relative change of ULF magnetic field at the ground

$$\frac{b_x}{b_{x0}}=\frac{b_{x0}-b_{x1}}{b_{x0}}=1-\varepsilon\frac{L}{\sqrt{\pi}}A(k_x)\int_{-\infty}^{\infty}dkB(k)\cdot$$
$$\cdot e^{i(k-k_x)x}\left\{e^{-(k-k_x)^2L^2}+\frac{1}{2}\left[e^{-(k_g-k+k_x)^2L^2-\varphi}+\right.\right.$$
$$\left.\left.+e^{-(k_g+k-k_x)^2L^2+\varphi}\right]\right\}$$
$$\varepsilon=\frac{\Delta\Sigma_{P0}}{\Sigma_A+\Sigma_{P0}},\quad A=\frac{k_1(k_x)+k_2}{k_x}\sinh(k_x h)$$
$$B=\frac{k}{\sinh(kh)\left[k_1(k)+k_H^2/k_2\right]}. \quad (3.6)$$

Finally after averaging we have for the value $\beta(\omega)=\sqrt{\langle|b_x/b_{x0}|^2\rangle}$ the following resultant relation:

$$\beta=1-\varepsilon\frac{\alpha}{\sqrt{\pi}}\mathrm{Re}\left\{A(\Omega,\varsigma_x)\int_{-\infty}^{\infty}d\varsigma B(\Omega,\varsigma)\cdot\right.$$
$$\left.\cdot\exp\left[i(\varsigma-\varsigma_x)\xi-\alpha^2(\varsigma-\varsigma_x)^2\right]\right\}$$
$$A=\frac{\sinh(\varsigma_x)}{\varsigma_x}\left(a_1\Omega+\sqrt{\Omega^2-\varsigma_x^2}+i\varsigma_x\coth(\varsigma_x)\right)$$
$$B=\frac{\varsigma}{\sinh(\varsigma)\left(a_2\Omega+\sqrt{\Omega^2-\varsigma^2}+i\varsigma\coth(\varsigma)\right)}$$
$$\xi=x/t,\quad \varsigma=kh,\quad \varsigma_x=k_x h=2\pi h/\lambda_x$$
$$a=L/h,\quad \Omega=\omega/\omega_A$$
$$\omega_A=c^2/4\pi\Sigma_A h=C_A/h,\quad a_1=1+2\frac{\Sigma_{P0}}{\Sigma_A}$$
$$a_2=\frac{\Sigma_{P0}}{\Sigma_A}+\frac{\Sigma_{H0}^2}{\Sigma_A(\Sigma_{P0}+\Sigma_A)}. \quad (3.7)$$

In (3.7) we denote undisturbed integrated Pedersen and Hall conductivities Σ_{P0}, Σ_{H0} respectively. In a case of the large perturbation zone, $\alpha>>1$ after simple calculations we have

$$\beta(\Omega,\xi)=1-\varepsilon\exp\left(-\frac{\xi^2}{4\alpha^2}\right)\mathrm{Re}\cdot$$
$$\cdot\left\{\frac{\varsigma_0\sinh(\varsigma_x)\left[a_1\Omega+\sqrt{\Omega^2-\varsigma_x^2}+i\varsigma_x\coth(\varsigma_x)\right]}{\varsigma_x\sinh(\varsigma_0)\left[a_1\Omega+\sqrt{\Omega^2-\varsigma_0^2}+i\varsigma_0\coth(\varsigma_0)\right]}\right\}$$
$$\varsigma_0=\varsigma_x+i\frac{\xi}{2\alpha^2} \quad (3.8)$$

In the center of the zone, $\xi=0$ relation (3.8) reduced to the following:

$$\beta(\Omega)=1-\varepsilon\frac{f_1 f_2+f_3^2}{f_2^2+f_3^2}$$
$$f_{1,2}=a_{1,2}\Omega+\sqrt{|\Omega^2-\varsigma_x^2|}\,\eta(\Omega-\varsigma_x)$$
$$f_3=\sqrt{|\Omega^2-\varsigma_x^2|}\,\eta(\varsigma_x-\Omega)+\varsigma_x\coth\varsigma_x \quad (3.9)$$

where $\eta(x\geq 0)=1$, $\eta(x<0)=0$.

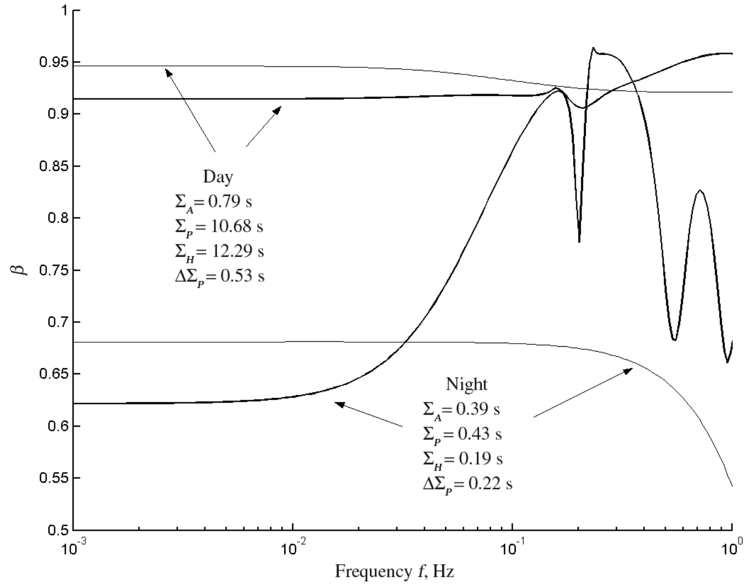

Fig. 2. Dependence of the relative magnetic field β at the ground on frequency for night and day hours. Thin lines show the results of calculations in the thin film ionospheric model, bold lines correspond to full-wave calculations in the IRI-90 model. $k_x = 0.01$ km^{-1} and other parameters are shown on the picture.

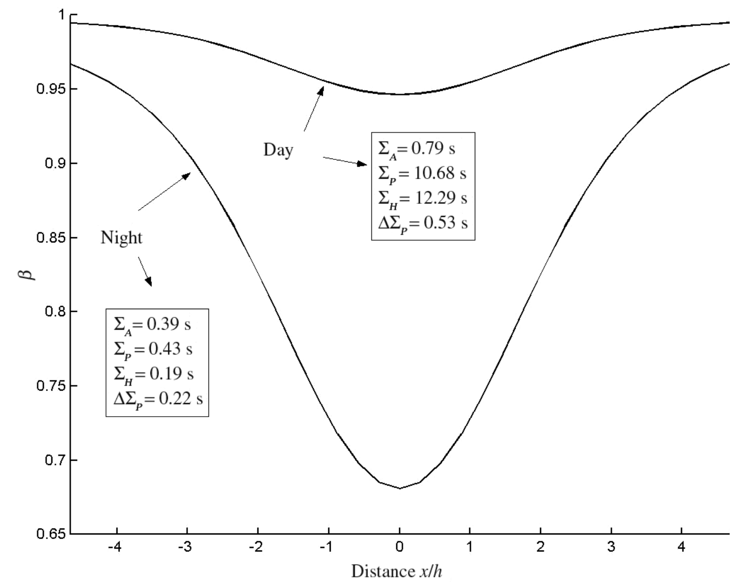

Fig. 3. Spatial distribution of the relative magnetic field β dependence on distance from projection of the perturbation center in the ionosphere, h is height of lower boundary of the ionosphere (here $h = 100$ km).

It is evident that $\beta(\omega = 0) = 1 - \varepsilon$, $\beta(\omega \to \infty) = 1 - \varepsilon\,(a_1 + 1)/(a_2 + 1)$ and $\beta(\omega)$ decreases on frequency. At night-time ionosphere when $\Sigma_{P0} << \Sigma_A$ we have $\varepsilon \approx \Delta\Sigma_{P0}/\Sigma_A$, $a_1 \approx 1$, and $a_2 << 1$. At day-time ionosphere when $\Sigma_{P0} >> \Sigma_A$ we have $\varepsilon \approx \Delta\Sigma_{P0}/\Sigma_A$, $a_1 \approx a_2 >> 1$. Dependence of β on frequency under the center of inhomogeneity is shown in fig. 2 for the night and day hours. The values of integrated Pedersen and Hall conductivities are given in the figure. For comparison, these dependences calculated for laterally homogeneous ionosphere of the finite thickness are also shown in fig. 2 for the same values of integrated ionospheric conductivities. The values of $\beta(\omega)$ at relatively low frequencies ($f < 0.05$ Hz) are about 0.6-0.7 for nighttime and 0.9-0.95 at daytime for both models. At higher frequencies (0.2-0.3 Hz) $\beta(\omega)$ calculated with the use of IRI model grows almost up to unity. Several extrema seen at the curve are caused by the ionospheric Alfven resonance and ionospheric MHD waveguide. This effect is not obtained in the thin ionosphere model that gives strong monotonous decrease of β with frequency at $f > 0.3$ Hz. At daytime β weakly depends on frequency and is about unity in all the frequency range considered. Spatial distribution of β value at the ground found with the relation (3.7) is presented in fig. 3. Note that the size of the depression area is larger than the perturbation scale in the ionosphere.

4. Discussion and conclusions

It seems that theoretical results here coincide with observational data reported by Molchanov *et al.* (2003). Recently Sorokin *et al.* (2002, 2003) investigated the possibility of generating seismo-induced geomagnetic pulsations due to preseismic changes in the background electric field.

Here we try to estimate another possibility in connection with intensification of the atmospheric gravity waves before earthquakes and following the appearance of the ionospheric inhomogeneities. While influence of the horizontal ionospheric irregularities on propagation of the VLF waves is known (*e.g.,* Shklyar and Nagano, 1998), the same influence on the Alfven waves is a rather original problem and we are going to continue this research for a more complicated model taking into consideration both vertical and horizontal stratifications of the ionosphere.

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Appendix

Let us represent the solution of (2.3) as the sum of downgoing and reflecting waves

$$E_x(k,z,\omega) = E_*(k)\left[e^{ik_A z} + R^a(k,\omega)\,e^{-ik_A z}\right]$$

$$b_y(k,z,\omega) = E_*(k)\left(c/C_A\right)\left[e^{ik_A z} - R^a(k,\omega)\,e^{-ik_A z}\right]$$

$$E_*(k) = \int_{-\infty}^{\infty} dx e^{-ikx} E_*(x), \quad k_A = \omega/C_A \tag{A.1}$$

where $E_*(x,\omega)$ is amplitude distribution on the wave front and R^a is reflection coefficient.

The similar relationship for isotropic wave as follows:

$$E_y(k,z,\omega) = E_*(k) R^i(k,\omega) e^{-ik_i z}$$
$$b_x(k,z,\omega) = \frac{k_i}{\omega} E_*(k) R^i(k,\omega) e^{-ik_i z} = \frac{k_i}{\omega} E_y(k,z,\omega)$$
$$k_i = \sqrt{k_A^2 - k^2} \qquad \text{(A.2)}$$

where R^i is reflection coefficient for isotropic waves. As concerned solution of (2.5) we need to take into consideration that $E_{x,y}(z = h) = 0$, and to match with above-mentioned solutions at the lower boundary of the ionosphere, hence

$$E_x(k,z,\omega) = E_*(k) T^a(k,\omega) \sinh\left[k(z-h)\right]$$
$$b_y(k,z,\omega) = -i\frac{\omega}{k} E_*(k) T^a(k,\omega) \cosh\left[k(z-h)\right] = -i\frac{\omega}{k} \coth\left[k(z-h)\right] E_x(k,z,\omega)$$
$$E_y(k,z,\omega) = E_*(k) T^i(k,\omega) \sinh\left[k(z-h)\right]$$
$$b_x(k,z,\omega) = i\frac{k}{\omega} E_*(k) T^i(k,\omega) \cosh\left[k(z-h)\right] = i\frac{k}{\omega} \coth\left[k(z-h)\right] E_y(k,z,\omega) \qquad \text{(A.3)}$$

where T^a is transmission coefficient for Alfven wave and T^i is coefficient of transformation the Alfven wave into isotropic wave in the ionosphere. It is evident that $b_y/b_x = (\omega/c)^2 T^a/(k^2 T^i) << 1$, if the transformation is essential. It means that geomagnetic pulsations observed at the ground surface are mainly related to isotropic wave, which is originated from Alfven wave inside ionosphere.

The electric fields at the upper boundary of the ionosphere are continuous, hence

$$1 + R^a = -T^a \sinh(kh), \quad R^i = -T^i \sinh(kh). \qquad \text{(A.4)}$$

However discontinuity of the magnetic fields equals to surface currents at the boundary

$$\int_{-\infty}^{\infty} dt e^{i\omega t} \int_{-\infty}^{\infty} dx e^{-ikx} b_x(x, z=+0, t) - \int_{-\infty}^{\infty} dt e^{i\omega t} \int_{-\infty}^{\infty} dx e^{-ikx} b_x(x, z=-0, t) =$$
$$= \frac{4\pi}{c}\left[\int_{-\infty}^{\infty} dt e^{i\omega t} \int_{-\infty}^{\infty} dx e^{-ikx} \Sigma_P(x,t) E_y(x,0,t) + \int_{-\infty}^{\infty} dt e^{i\omega t} \int_{-\infty}^{\infty} dx e^{-ikx} \Sigma_H(x,t) E_x(x,0,t)\right]$$
$$\int_{-\infty}^{\infty} dt e^{i\omega t} \int_{-\infty}^{\infty} dx e^{-ikx} b_y(x, z=+0, t) - \int_{-\infty}^{\infty} dt e^{i\omega t} \int_{-\infty}^{\infty} dx e^{-ikx} b_y(x, z=-0, t) =$$
$$= -\frac{4\pi}{c}\left[\int_{-\infty}^{\infty} dt e^{i\omega t} \int_{-\infty}^{\infty} dx e^{-ikx} \Sigma_P(x,t) E_x(x,0,t) - \int_{-\infty}^{\infty} dt e^{i\omega t} \int_{-\infty}^{\infty} dx e^{-ikx} \Sigma_H(x,t) E_y(x,0,t)\right] \qquad \text{(A.5)}$$

Suppose now that time variation of the ionospheric conductivity is slow in comparison with geomagnetic pulsations. For example characteristic frequencies of the atmospheric gravity waves are $\omega_0 \sim (10^{-3} - 10^{-4})c^{-1} << \omega$. If so, we can simplify time integration in (A.5). Using now (2.1), (A.1-A.4) and obvious relation

$$\int_{-\infty}^{\infty} dt e^{i\omega t} \int_{-\infty}^{\infty} dx e^{-ikx} \Sigma_{P,H}(x,t) E_{x,y}(x,0,t) = \int_{-\infty}^{\infty} dx e^{-ikx} \Sigma_{P,H}(x,t) E_{x,y}(x,0,\omega) =$$

$$=\Sigma_{P,H0}E_{x,y}(x,0,\omega)+\int_{-\infty}^{\infty}\frac{dk'}{2\pi}\Delta\Sigma_{P,H}(k-k',t)E_{x,y}(k',0,\omega) \tag{A.6}$$

we finally find for electric field in the ionosphere

$$\widehat{K}(k,\omega)\boldsymbol{E}(k,0,\omega)=2k_A\boldsymbol{E}_*(k)-\frac{4\pi\omega}{c^2}\int_{-\infty}^{\infty}\frac{dk'}{2\pi}\Delta\widehat{\Sigma}(k-k',t)\boldsymbol{E}(k',0,\omega). \tag{A.7}$$

Here time t is parameter and definitions are as follows:

$$\boldsymbol{E}=\begin{bmatrix}E_x\\E_y\end{bmatrix},\quad \boldsymbol{E}_*=\begin{bmatrix}E_*\\0\end{bmatrix},\quad \widehat{K}=\begin{bmatrix}k_1 & -k_H\\k_H & k_2\end{bmatrix},\quad \Delta\widehat{\Sigma}=\begin{bmatrix}\Delta\Sigma_P & -\Delta\Sigma_H\\\Delta\Sigma_H & \Delta\Sigma_P\end{bmatrix}$$

$$k_1=k_P+k_i+ik\coth(kh),\quad k_2=k_P+k_A,\quad k_{P,H}=4\pi\Sigma_{P,H}\omega/c^2$$

$$k_A=4\pi\Sigma_A\omega/c^2,\quad \Sigma_A=c^2/4\pi C_A.$$

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